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Income Taxes, Life-Cycles and Growth

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Abstract

A numerical, two-sector, three-period, overlapping generations model is used to investigate and quantify the effects of income taxes on economic growth. This paper stresses the role of life-cycle effects the determining growth. In this model, growth is generated by the accumulation of physical and human capital. Two different specifications for the human capital technology and three different specifications for leisure activity are examined; therefore, this paper explores six different models. The consideration of life-cycle effects further distinguishes human capital from physical capital. The paper shows that life-cycle effects are always important in determining growth and may dominate all other effects in some models. Moreover, the income tax discriminates against investment in physical capital in all six models. The simulation results show the extent to which growth and output are affected by the specifications chosen for the human capital technology and the leisure activity. Asymmetric technologies in the two sectors lessen growth effects but strengthen intersectoral reallocations, while an elastic labour supply strengthens growth effects. The outcomes of the simulation experiments are also different, in many ways, from those found in the literature utilizing unique representative agent (Ramsey) models.

Résumé

Les effets de la taxation à la marge des revenus sur le taux de croissance économique sont analysés et quantifiés à l'aide d'un modèle numérique de trois générations imbriquées et deux secteurs de production. L'étude fait ressortir l'importance des phénomènes reliés au cycle de vie dans la détermination de la croissance économique. C'est l'accumulation du capital physique et humain qui génère la croissance dans le modèle. Deux spécifications pour la technologie reliée au capital humain et trois spécifications pour le loisir sont explorées. Six différents modèles sont donc analysées dans cette article. La prise en considération des phénomènes de cycle de vie permet de distinguer davantage le capital humain du capital physique. L'article démontre que les effets reliés au cycle de vie sont toujours importants dans la détermination de la croissance et peuvent même dominés tous les autres effets dans certaines spécifications étudiées. Pour les six modèles, les résultats de simulations rapportent que la taxe sur les revenus discriminent contre l'investissement en capital physique. Les simulations montrent aussi comment les effets sur la croissance et l'output économiques sont dépendants des spécifications choisies. Une spécification avec des technologies de production asymmétriques amenuise les effets sur la croissance, alors qu'une spécification du loisir qui accroit l'élasticité de l'offre de travail les renforce. Enfin, l'article relève les différence dans les résultats de ceux qui sont obtenus avec les modèles à agent représentatif unique (modèles à la Ramsey).

1. INTRODUCTION

The endogenous growth literature, which first appeared in two seminal papers by Romer (1986) and Lucas (1988), has since expanded at a higher rate than most economies.[1] Two main approaches have been taken. One approach, which focuses on invention, innovation, imitation, (Romer 1990; Grossman and Helpman 1991; Chou and Shy 1991; Aghion and Howitt 1992) and absorption (Keller 1994), needs imperfect competitive markets to motivate private profit-seeking investment in knowledge. Total factor productivity is the main source of growth, meaning that income gains generated by higher productivity are external to the economic activities that generate growth. The activities of improving productivity and of producing commodities are connected when the benefits created from the former sector (R&D for instance) "spill over" to income recipients of the latter activity. The second approach differs from the first by incorporating the sources of growth entirely within the production activities. Total factor productivity is nil, growth is generated by the accumulation process, and it is the private agents who internalize the returns from capital accumulation activities. Although the accumulation process is compatible with perfectly competitive markets and zero total factor productivity growth, it is incompatible with zero marginal products in the accumulating factors (Jones and Manuelli 1990, Mulligan and Sala-i-Martin 1993). Constant positive returns in the accumulating factors generate a balanced growth path that is compatible with the stylized facts of economic growth described by Kaldor (1963). Alternatively, when the balanced growth path is estimated to be too far in the future as in Jorgenson (1994), growth occurs during the -infinite- transition path, and is ascribed to the substitution process (in the production) rather than to the accumulation process.

It may be deduced that one part of growth stems from the process of accumulation and/or substitution, the other part from total factor productivity increases. Since the first part has been found to be relatively large in empirical studies,[2] it is relevant to evaluate how the process of accumulation and substitution is affected by tax policies. This paper investigates and quantifies how economic growth (or more specifically how the accumulation of physical and human capital) is affected by marginal income tax distortions when the accumulation processes are influenced by life-cycle considerations.

Many of the contributions in the literature, on the issue of the effect of taxation on growth, have been at the theoretical level (Jones and Manuelli 1990, Liu 1994, Mino 1996). The few quantitative studies that have been done yield quite different results. King and Rebelo (1990), Pecorino (1993), Jones, Manuelli, and Rossi (1993), and

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[1] A summary of the recent contributions in this literature can be found in the symposium of The Journal of Economic Perspectives (Spring 1994), and in Barro and Sala-i-Martin (1995).

[2] In the traditional growth-accounting sense, Young (1994) attributes all of the East Asian growth experience to the sole result of factor accumulation. Jorgenson (1994) attributes to factor substitution 83% of US growth over the period 1948-84.

Devereux and Love (1994) report a strong negative effect of income taxes on the long-run growth rate.[3] However, Kim (1992) and Lucas (1990) find only moderate and weak effects, respectively. All of the above studies use the representative agent (Ramsey) framework in a two-sector model, in which growth is generated by the accumulation of physical and human capital. Stokey and Rebelo (1995) investigate the sources of the sharp differences in the growth effects, and find that the results are sensitive to the assumptions regarding the production function of human capital, the depreciation rate of capital and its tax treatment, and the elasticity of labour supply.

The role played by the specification of the human capital production function in the different growth results is related to the question concerning the extent to which human capital differs from physical capital. In a two-sector growth model, a tax change will generate both intersectoral and intertemporal reallocations. The greater the intertemporal reallocation is, relative to the intersectoral reallocation, the higher will be the growth effect of a new tax policy. When the specification of the human capital production function is symmetric to that of physical capital,[4] tax policies have qualitatively similar effects on both accumulating activities. For example, in a Ramsey framework, where the specification of the human capital production function is symmetric to that of physical capital "an increase in the income tax rate decreases the rate of return to the investment activities of the private sector and leads to a permanent decline in the rate of capital accumulation and in the rate of growth" (Rebelo 1991, p. 501). In that case, an income tax increase produces an intertemporal effect (future versus current consumption) generalized to all investment activities.[5]

When the production function specifications differ, there is more scope for intersectoral reallocation and sector-specific intertemporal substitution effects. A new tax policy may generate more intersectoral than intertemporal reallocation, which weakens its growth effects. In Lucas (1990), for instance, human capital is the only input in the production of future human capital. The growth effects of a change in the tax rate on factors of production are small in this model. The sector-specific intertemporal effect may also be negatively correlated across sectors. For instance, Buiter and Kletzer (1991,

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[3] Although not used for that purpose, the increasing returns to scale model of Romer (1986) would also generate strong growth effects.

[4] Symmetric production functions are those that depend on the same inputs and have the same elasticity of substitution.

[5] In a Ramsey framework, where the production functions for the accumulating factors are modeled symmetrically and labour supply is inelastic, the model can be solved recursively: the gross input factor prices ratio is determined by the supply side of the economy and the interest rate net of tax determines the growth rate, given the preference parameters. In this context, an increase in the flat-rate income tax reduces proportionally the net of tax interest rate and the effect on the long-run growth rate is significantly negative. To be precise, Rebelo (1991) and Stokey and Rebelo (1995) show that the long-run change in the growth rate of the economy corresponds to the change in the net interest rate times the intertemporal elasticity of substitution.

1993) build a model in which what is bad for one kind of investment is good for another. The overall effect of the tax change on growth in their framework then depends on which type of investment matters most for growth.

In a Ramsey framework, human capital can also differ from physical capital (see Milesi-Ferretti and Roubini 1994) by being not substitutable with consumption and non-market goods. While human capital is not substitutable with consumption, it can be substitutable with leisure if both activities share the property of being time dependent. An elastic labour supply function then provides another avenue through which agents can respond to changes in net investment returns, and thus strengthens the tax effect on growth (as in Jones, Manuelli and Rossi 1993). The non-marketability of human capital implies that some inputs used for the production of human capital are not subject to factor income taxation.[6] Asymmetric tax burdens across sectors favours intersectoral reallocation and sector-specific intertemporal substitution effects. To recapitulate, symmetric production functions and symmetric tax burdens, combined with an elastic labour supply, lead to stronger growth effects of tax policy in Ramsey models.

This paper intends to add a fourth relevant dimension to human capital: the fact that human capital is embodied in people who have a finite lifetime. This new dimension has nothing spectacular at first sight, and was recognized a long time ago in the literature (Mincer 1974; Heckman 1976; Haley 1976). However, it is noteworthy to recall that with this dimension, sales of the stock of human capital are excluded unless slavery is permitted, and that both the principal and the returns to investment in human capital fall to zero at retirement for the individual who has accumulated it.[7] The absence of a market for the stock of human capital implies that the benefit from investing in human capital is the stream of future revenues from labour supply. The earlier the investment is realized in age, the higher will be the benefit. This, combined with the embodiment and finite lifetime properties, involves important life-cycle considerations. For example, the intertemporal trade-off is not between current and next period consumption, but between current and the rest-of-life consumption. With such a trade-off, current period investment decision depends upon previous decisions and the entire future payoff. The presence of a competitive market for physical capital implies that investors are return-takers in this market; the absence of a market for human capital individualizes its production, and consequently empowers the same investors as return-makers for this asset. With return-makers as investors in human capital there is more scope for intersectoral reallocation and sector-specific intertemporal substitution effects. A full account of the embodiment dimension in the discussion of income tax policy requires a model with finite horizon representative agents, heterogeneous in their marginal propensity to save in the available assets. Unlike the present literature, an Allais-Samuelson overlapping generations model

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[6] This explains why in many countries there are higher tax rate differences across sectors of activity than across factors of production.

[7] If human capital generates a non-pecuniary value, the 100% depreciation rate will occur at death.

(OLG) has been adopted for the income tax numerical analysis. To allow an accumulation of human capital at the aggregate level and to ensure the existence of a balanced growth path,[8] the model contains an externality which transfers to future generations a fraction of the stock of human capital accumulated by all living generations.[9]

There is a common view in the literature which claims that although the Ramsey and OLG models have very different theoretical frameworks, they yield, in practice, quite similar results for tax problems.[10] This view, and the fact that the Ramsey structure is analytically more tractable, explains why so few tax investigations on growth have been done with OLG models.[11] However, this view does not hold for endogenous growth models. Jones and Manuelli (1992) show that the observational equivalence between overlapping generations and infinitely lived models, in the sense of OLG models replicating the equilibrium path of infinite horizon models, does not hold when growth is endogenously driven. This paper shows that life-cycle issues significantly influence the growth results. Moreover, since each contemporary agent has a different finite planning horizon in an OLG framework, marginal propensities to spend and save will differ across agents. As a result, even if the long-run effects with an OLG structure were similar to those obtained with a Ramsey structure, the adjustment path of such economies to various shocks would differ from the adjustment path of economies with agents that have infinite planning horizons.

The rest of this paper is organized as follows. Section 2 presents the general model. Section 3 discusses life-cycle issues, and the role that the different specifications adopted for human capital technology and for leisure activity should play in the growth results. Section 4 outlines the calibration procedure. Section 5 reports and discusses the results. Section 6 provides concluding remarks.

2. THE GENERAL MODEL

To consider the various dimensions of human capital, a three-period two-sector overlapping generations framework is necessary at a minimum. The population is thus composed of young, middle-aged and old generations, while production takes place in the final goods sector (future physical capital) and in the education sector (future human

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[8] As first highlighted by Jones and Manuelli (1990).

[9] Sengupta (1991), in an empirical analysis, concludes that South Korean economic growth has been significantly helped by the sizable impact of human capital spillovers.

[10] See for instance the discussion in Lucas (1990).

[11] Liu (1994) presents a theoretical analysis with two-period OLG set up. The main reference for tax policy investigation using a numerical OLG model is Auerbach and Kotlikoff (1987). Growth is however exogenous in this important study.

capital). With three-period overlapping generations and two state variables, the tractability of the model is cumbersome. A numerical algorithm is developed in this paper to solve the model for the stationary as well as the transitory paths. Devereux and Love (1994) show how important the transitional effects can be in evaluating the overall impact of taxes on growth.

Since the specifications for human capital production and labour supply have been shown to be crucial for the growth results in Ramsey models (Stokey and Rebelo 1995), the life-cycle considerations of taxes on growth are assessed in six different cases. Two different production technologies for human capital are examined. The first has only human capital and educational time as inputs (Lucas 1988, 1990), and is called the H specification. The second has human capital, educational time, and a flow of consumption goods as inputs (Ben Porath 1967, Trostel 1993), and is called the HK specification. To investigate different degrees of substitution between leisure and human capital activities, and hence different degrees of labour supply elasticity, three different specifications for the leisure activity are examined. The first is the exogenous labour supply specification: time allocation to the leisure activity is nil the first two periods of life, but full at the third period. This specification is denoted by EL. The second is the raw time (Lucas 1990; Jones, Manuelli, and Rossi 1993) specification: leisure depends on the fraction of time allocated to that activity. This specification is denoted by RT. The third is the quality time (Becker 1965; Heckman 1976) specification: leisure activity is the product of the fraction of time allocated to it and the stock of human capital accumulated. This specification is denoted by QT. With two different specifications for human capital technology and three different specifications for leisure activity, there are a total of six different models in this paper, which are denoted by EL-H, RT-H, QT-H, EL-HK, RT-HK, and QT-HK.

Whereas the economy is composed of two sectors of production, only the factors used in the physical capital sector are taxed at equal flat-rates. Tax payments are assumed rebated in a lump-sum manner, so the analysis is limited to the effect of marginal tax distortions.[12] Without loss of generality, population growth is assumed to be zero. The structural equations for the production of physical capital will be first described. The structural equations for the production of human capital will follow, then the optimization problem of a representative individual, and finally the equilibrium conditions will be outlined.

2.1. The production of physical capital

The final goods sector production depends on physical capital and effective labour. All firms are identical. As first proposed by Tinbergen (1942) and corroborated by

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[12] The effects of the redistribution of tax revenues among the different generations are investigated by Jones and Manuelli (1992) and Liu (1994).

Jorgenson *et al.* (1987)[13], the aggregate production function is a useful simplification. The technology is assumed to take the Cobb-Douglas form:[14]

$$Y_t = AK_t^{\varepsilon} L_{e,t}^{1-\varepsilon} \quad , \tag{2.1}$$

where Y is final goods output, K is physical capital and L_e is effective labour. Physical capital can be accumulated as forgone consumption, which is equivalent to assuming that physical goods are produced in a separate sector that has the same technology as the final goods sector. Assuming zero capital depreciation, denoting aggregate consumption by C and the flow of goods invested in the education sector by S_h,[15] the accumulation motion for physical capital is derived from the familiar equilibrium condition in the final goods sector :

$$K_{t+1} - K_t = Y_t - C_t - S_{h,t} \quad . \tag{2.2}$$

In the above equation, government expenditures are nil since tax payments are rebated lump-sum to consumers.

Factor demands stem from profit maximization by the firms. Firms rent physical capital from current generations at the rental rate r and hire labour at the wage rate w per unit of effective labour, up to the point at which their marginal products equal their marginal costs:

$$r_t = \varepsilon A K_t^{\varepsilon-1} L_{e,t}^{1-\varepsilon} \quad , \tag{2.3}$$

$$w_t = (1-\varepsilon) A K_t^{\varepsilon} L_{e,t}^{-\varepsilon} \quad . \tag{2.4}$$

Assuming the existence of a consumption-loan asset, perfectly substitutable to physical capital, a standard no-arbitrage condition implies that the physical rental rate is also the interest rate in the economy. Denoting the interest tax rate and the wage tax rate by τ_r and τ_w respectively, the net interest rate and the net wage rate can thus be represented as:

$$\tilde{r}_t = (1-\tau_{r,t}) r_t \quad , \tag{2.5}$$

$$\tilde{w}_t = (1-\tau_{w,t}) w_t \quad . \tag{2.6}$$

2.2. The production of human capital

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[13] They estimate that errors arising from aggregation over industrial sectors explained less than 3% of aggregate productivity growth over the period 1948-1979 in the United States.

[14] Not much is lost by sticking to a Cobb-Douglas production function. Stokey and Rebelo (1995) find that the elasticities of substitution in production are rather unimportant for the quantitative impacts of tax reform.

[15] This flow of goods will be different of zero only in the three HK models.

2.2.1. *The accumulation motion of the human capital asset*

The education sector is described by a well-behaved technology, linear with respect to human capital and the flow of final goods, but strictly concave with respect to educational time. Human capital production is individualized, and total production of new human capital in the economy is simply the sum of all generations' production. Since there are two specifications investigated in this paper, a generation born at time t faces one of the following accumulation motions:

$$h_{t+1} = \frac{h_t}{1+\delta} + Bh_t z_t^{\gamma} \quad , \tag{2.7}$$

$$h_{t+1} = \frac{h_t}{1+\delta} + Bh_t^{1-\alpha} s_{h,t}^{\alpha} z_t^{\gamma} \quad , \tag{2.8}$$

where h is the embodied human capital stock, z is the fraction of time allocated to education, s_h is the flow of goods invested in the education sector, and δ is the human capital depreciation rate. The parameters of the functions are subject to the following restrictions: $B > 0; 0 < \alpha < 1; 0 < \gamma < 1; \alpha + \gamma < 1; \delta > 0$. The specification in (2.7) is similar to that in Lucas (1988, 1990). The specification in (2.8) is similar to that in Jones, Manuelli, and Rossi (1993). King and Rebelo (1990) and Kim (1992) use the stock of capital rather than the flow of goods as inputs. Whether s_h is a flow or a stock does not matter. What matters is that s_h originates from the production of the final goods sector, and is thus a taxable factor of production. The assumption of a positive depreciation rate for human capital, but of a zero depreciation rate for physical capital, reflects the fact that the latter is commonly tax deductible while the former is not, as pointed out by Nerlove *et al.* (1993). Both functions in (2.7) and (2.8) are linearly homogenous with respect to h and s_h. However, both point-in-time technologies are non-linear in the choice variables s_h and z. The plausible assumption that educational output depends mainly on students' effort is translated into an extreme specification, in which only students' effort (educational time and personal expenditures) is necessary to have a positive production in the education sector. Jorgenson (1994) supports such specifications. A relaxation of these assumptions, which would preserve the factor intensity orderings, would not likely change the basic dynamics of the model.

2.2.2. *The social heritage mechanism*

Because of its embodiment property, the stock of accumulated human capital depreciates to zero at death or retirement from the investor's perspective. An intergenerational transfer mechanism is needed in order to make human capital a reproducible -or accumulating- asset for the economy. That mechanism will be called the social heritage mechanism (SHM). Through it, the initial human capital stock of a new

born generation h_t^1 is a linear function of the human capital stock accumulated by all living generations of the previous period:

$$h_t^1 = \pi \sum_{g=1}^{G} h_{t-1}^g \quad , \tag{2.9}$$

where h_{t-1}^g is the stock of human capital accumulated at period *t-1* by a generation of age *g*; π is a constant parameter, and G is the number of lifetime periods and living generations. This social heritage mechanism allows any new generation to benefit to some extent (determined by π) of the investments made by older and previous generations. It is an automatic social bequest, involuntarily made by the elders. The mechanism allows an infinite process of accumulation of human capital in the economy, even though this process is finite for each generation.

The heritage mechanism introduces a non-rival, non-exclusive good into the model. Non-rival because all members of a new generation benefit to the same extent from previous investments made in human capital, whatever the population growth rate. Non-exclusive because old generations cannot be paid off by members of the new born generation for this transfer, since the necessary market is missing. An example of this social heritage mechanism can be drawn from economic knowledge: graduate macroeconomic and microeconomic course materials are updated regularly; all new students benefit from the progress made in the discipline of economics, but the admission fee for economic studies is independent of this updating process.

Through the SHM, newer generations are able to consume and save more in their first period of life than their ancestors. Since young generations can accumulate ever increasing amounts of (both) capital, growth occurs in the economy. Technically speaking, Appendix I shows that with the SHM, the human capital accumulation motion for the economy is linearly homogenous with respect to accumulating inputs. This, combined with the constant returns to scale technology in accumulating factors for physical capital, ensures that the marginal product of the broad (physical and human) capital is bounded above zero in the economy, which is sufficient for the existence of an endogenous balanced growth path.[16]

2.2.3. The accumulation motion of the physical capital asset

The representative generation's budget constraint at each period is:

$$a_{k,j+1} - a_{k,j} \leq \tilde{w}_t h_t (1 - z_t - l_t) + \tilde{r}_t a_{k,j} - c_t - s_{k,j} - s_{h,j} + tr_t \quad , \tag{2.10}$$

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[16] On the conditions for endogenous growth in finite life economies, see Jones and Manuelli (1992).

where a_t is the accumulated physical capital asset rented to the firms and s_j (j=k,h) are respectively savings in physical capital and in education. The rebated lump sum tax payments is represented by tr, whereas the fraction of time allocated to leisure is denoted by l. Note that the rate of return from investing in physical capital faced by the representative generation is given and equal to the rate of interest net of taxes. However, the rate of return from investing in human capital is absent in this equation since this rate is not determined by a market, but is determined by educational time (z).

2.2.4. Preferences

The generations' preferences are represented by a constant intertemporal elasticity of substitution utility function that takes the following form:

$$U = \frac{\sigma}{\sigma - 1} \sum_{i=0}^{G} \left(\frac{1}{1+\rho} \right)^{i} \left[\left(c_{t+i}^{1-\theta} L_{t+i}^{\theta} \right)^{\sigma - 1/\sigma} - 1 \right] , \qquad (2.11)$$

where c is consumption, L is leisure activity, ρ the pure rate of time preference and σ the constant intertemporal elasticity of substitution. Leisure activity takes the following form:

$$L_t = h_t^{\phi} l_t \ . \qquad (2.12)$$

The functional form of the utility function is consistent with the existence of a balanced growth path as shown by King, Plosser and Rebelo (1988). When leisure does not generate utility (EL models), $\theta = 0$; when leisure has the raw time specification (RT models), $\phi = 0$; and when leisure has the quality time specification, $\phi = 1$. Under raw and quality time specifications, $0 < \theta < 1$.

2.3. The Optimization Problem of the Representative Individual

Every generation is rational. An individual born at time t takes its initial human capital stock h_t^1 as given and maximizes its utility function (2.11) subject to: the budget constraint (2.10), the accumulation motion of human capital (2.7 or 2.8), the leisure activity specification (2.12) and a one unit time endowment. At any age, the representative individual allocates a specific proportion of its time endowment toward each of leisure, work, and education. It has also to allocate its disposable income toward consumption and savings.

From an individual's perspective, the budget constraint (2.10) and the production technology (2.7) or (2.8) form a production possibility frontier. Equation (2.10) implies a linear point-in-time technology for physical capital, whereas (2.7) or (2.8) entails a non-linear point-in-time technology for human capital. Therefore, the production possibility frontier is strictly concave. This ensures interior solutions.

2.4. Equilibrium and Aggregation Conditions

There are three well-defined markets in the model. Three equilibrium conditions are thus needed to close the model. The final goods market equilibrium condition is represented by equation (2.2). The next two equations represent the equilibrium equations for effective labour and physical capital respectively:

$$L_{e,t} = \sum_{g} h_t^g \left(1 - z_t^g - l_t^g\right) , \tag{2.13}$$

$$K_t = \sum_{g} a_{k,t}^g \quad . \tag{2.14}$$

Note there is no intragenerational transfer of human capital. For living generations, human capital is a rival good and each worker cannot use the human capital stock held by others. Permitting such contemporary externalities would introduce global increasing marginal productivity of human capital and a competitive equilibrium would not exist in this set up.[17]

The economy will exhibit a balanced growth path when aggregate consumption, aggregate physical capital, and aggregate human capital grow at the same rate. On this balanced growth path new generations repeat the same choices as their ancestors. That means that the life-cycle shapes of individual consumption expenditures and savings profiles are repeated over and over. Similarly, time allocations (l and z) over the life-cycle remain constants. In other words, each new generation starts with a higher endowment of human capital (if growth is positive), but faces the same relative prices. It would thus choose the same consumption-income ratio patterns and same time allocation as its ancestors. But, because of its higher endowment in human capital, it would enjoy a higher standard of living for each period of life.

The OLG framework used in this analysis comprises six specific models. The next section discusses life-cycle issues, as well as the implications of the two specifications for human capital technology and of the three specifications for leisure activity.

3. LIFE-CYCLES, PRODUCTION AND LEISURE SPECIFICATIONS

3.1. Life-cycle considerations

When one moves from a Ramsey framework to an overlapping generations approach, two important features are added to two-sector endogenous growth models. The first originates from the life-cycle incidence of the distinct properties of physical and human capital. For instance, it has been argued that because of the finite lifetime and

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[17] See Romer (1986) for the necessary conditions needed on the investment function to permit competitive equilibrium.

embodiment properties of human capital, the effective depreciation rate from the investor's perspective jumps sharply toward the end of life (at retirement for the EL and the RT specifications; at death for the QT specification). These consequences contrast sharply with physical capital, since the existence of a competitive market precludes any life-cycle incidence to be reflected in physical capital depreciation rate. This and other distinctions between physical capital and human capital enlarge the scope for intersectoral and sector-specific intertemporal reallocations.

To understand how output and growth are affected by the scope for intersectoral and sector-specific intertemporal reallocations, Figure 1 sketches the stationary equilibrium of the two-sector model. The vertical axis represents the rate of accumulation of human and physical capital, and the horizontal axis --the physical-human capital ratio. A high physical-human capital ratio implies a low return for physical capital, because of the law of diminishing marginal productivity. Thus, the schedule for the accumulation rate of physical capital *dK/K* is downward sloping. The schedule for the accumulation rate of human capital *dH/H* is upward sloping for the same reason.[18] Balanced growth is achieved at the intersection of both schedules, where both accumulated inputs (and the economy) grow at the same rate, and the physical to human capital ratio remains constant.

In Figure 1, the assumption is made that technologies and sectoral tax burdens are symmetric in both sectors. In a Ramsey framework, a comprehensive and proportional rise in income taxes does not distort the trade-off between investments in physical capital and human capital. The path of the economy moves from point A to B with an unchanged physical-human capital ratio, but with a lower balanced growth rate, due to negative intertemporal effects. Nerlove *et al.* (1990, 1993) have shown that equal flat-rate taxes on labour and interest incomes distort the trade-off between investment in physical capital and human capital, when investors' lifetime is finite. It is further shown in Appendix III, that in an OLG framework that captures the finite lifetime and the embodiment properties of human capital, the trade-off between the two forms of investment is always distorted after a marginal change in income taxes. The Appendix demonstration implies that in Figure 1, the stationary equilibrium may move to point C if the distortion is against investment in human capital, but to point D if the distortion is against investment in physical capital. Notice that for the latter case, the effect on growth is positive since the upward shift of the *dH/H* schedule is large enough. The direction of the change in the stationary equilibrium depends on the specification of the production function, as discussed in the next sub-section.

The second feature is related to the redistribution of the tax burden. Since the returns to human capital are the discounted sum of future wage revenues, it is rational to invest in the education when young. Since the principal of the physical asset can be sold, it is rational to prefer this asset for old age retirement preparation. So the young generations invest mostly in human capital, and the middle-aged invest mostly in physical capital. As

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[18] See appendix II for a more rigourous development of this geometric exposition.

human capital differs from physical capital, a change in tax policy implies a shift of the tax burden across the age-heterogeneous agents.[19] The inference for aggregate outcomes becomes more intricate. The aggregate outcome depends on the number of people in each generation and on the tax burden shift across generations. Such factors are totally neglected in a Ramsey model.

3.2. The specification of the production functions

Consider again the two-sector model with asymmetric production functions in one case, and symmetric production functions in another.

Case 1. Asymmetric Production Functions

In this first case, the production of future physical capital depends on both physical capital and human capital inputs, whereas the production of future human capital depends only on human capital and a non-accumulating factor denoted by Z. The accumulation equations of the economy take the following forms:

$$\dot{K} = A(K,H) - C \ , \tag{3.2.1}$$

$$\dot{H} = B(H,Z) \ , \tag{3.2.2}$$

where both A and B functions are neo-classical and satisfy the Inada conditions. The growth process in this economy is endogenized by specifying the functions A and B appropriately. Function A is assumed linearly homogeneous in physical and human capital inputs, whereas function B is linear in the human capital input. Notice that with these specifications, human capital is essential for the future production of both types of capital. Faster accumulation of human capital accelerates the accumulation of physical capital by raising its marginal productivity. Faster accumulation of physical capital reallocates the current human capital stock from the education to the final goods sector for the same reason, but does not necessarily accelerate the accumulation of human capital. Thus, human capital should be considered the real engine of growth in this economy.

To illustrate this further, the above equations are rewritten using the linear homogeneity property common to both functions:

$$\frac{\dot{K}}{K} = A\left(1, \frac{K^{-1}}{H}\right) - \frac{C}{K} \ , \tag{3.2.3}$$

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[19] See Auerbach and Kotlikoff (1987) for a detailed discussion of intergenerational transfers of the tax burden.

$$\frac{\dot{H}}{H} = B\left(1, \frac{Z}{H}\right) . \qquad (3.2.4)$$

As long as Z is weakly sensitive to change in factor prices, the accumulation process of human capital is insensitive to the K/H ratio. The resulting *dH/H* schedule is horizontal, as illustrated in Figure 2. In this Figure, a rise in the marginal income tax on physical returns shifts the *dK/K* schedule to the left because of the negative intertemporal substitution effect on savings. That is, for any level of income (K/H) the private sector now accumulates less physical capital than before. However, the *dH/H* schedule remains intact as long as Z is insensitive to changes in factor prices. The balanced growth path moves from point A' to point B' in Figure 2. On the new balanced growth path, the ratio (K/H) is reduced, but not the accumulation rate of both types of capital. Assuming that Z is sensitive to factor prices through adverse sector-specific intertemporal effects, as in Buiter and Kletzer (1993). The *dH/H* schedule then shifts up and the new stationary equilibrium point C entails a higher balanced growth rate in the economy. Under both assumptions, however, level effects (changes in K/H ratio) are expected to be stronger than growth effects.

Case 2. Symmetric Production Functions

Suppose now that the production of future human capital requires also a flow of consumption goods as input. Denoting savings for producing human capital by S^h, the accumulation equation for human capital now takes this form:

$$\frac{\dot{H}}{H} = B\left(S^h, H\right) \qquad (3.2.5)$$

The function E is assumed linearly homogeneous with respect to S^h and H. Since the technology for producing future human capital depends on both savings and human capital, the schedule for the accumulation of human capital has, this time, a positive slope. Although factors producing human capital are not taxed directly, the physical savings input is taxed indirectly. With symmetric production functions, there is less scope for sector-specific intertemporal substitution effects. One must expect that a change in taxes would generate a greater change in the accumulation rate than in the previous case. Consequently, the impact on the K/H ratio would be more moderate, whereas the impact growth would be more important than in the first case.

The discussion related to Figures 1 and 2 indicates that the level and growth effects of a rise in income taxes depend in large part on the sensitivity of time allocation with respect to the production of human capital. In the next subsection, that sensitivity is discussed through the derivation of the solution of educational time z for the EL-H and EL-HK models.

3.3. Substitution effects in models with inelastic labour supply

In EL models, the investment decision concerning human capital accumulation does not directly involve any argument of the utility function. The optimization of intertemporal income can thus be separated from the utility maximization problem. To grasp the effect of income taxes on the accumulation of human capital, the interior solution for educational time are next derived for the EL-H and EL-HK models, respectively.

The EL-H model

In the EL-H model, the first step problem consists in maximizing the following equation:

$$\tilde{w}h_t(1-z_t)+\frac{\tilde{w}h_{t+1}(1-z_{t+1})}{1+\tilde{r}}+\frac{\tilde{w}h_{t+2}(1-z_{t+2})}{(1+\tilde{r})^2}$$

subject to:

$$h_{t+1+i}=\frac{h_{t+i}}{1+\delta}+bh_{t+i}z_{t+i}^{\gamma}\ ,\ i=0,1.$$

Since the representative generation does not work in its third period of life and since human capital does not enter into the utility function, $z_{t+1}=z_{t+2}=0$. Manipulation of the first-order conditions then gives the following solution for z_t:

$$z_t=\left[\frac{b\gamma}{1+\tilde{r}}\right]^{1/1-\gamma}\ . \qquad (3.3.1)$$

Note that the real wage rate is not an element of the solution. Since any change in the wage rate affects costs and benefits of investment in human capital in the same proportion, the optimal solution for z is independent of the wage tax rate. A rise in interest income taxes implies a lower net interest rate,[20] and hence stimulates investment in human capital by raising the present discounted value of the stream of revenue from future labour supply. The lower net of tax interest rate, however, discourages investment in physical capital. This discrimination in favour of human capital and against investment in physical capital enhances growth, because the growth process is driven by the accumulation of human capital. This partial equilibrium solution suggests a possible move of the stationary equilibrium from point A to point D in Figure 1. Although the endogenization of leisure choices would complicate the analysis, it does suggest at this point an important intersectoral reallocation and sectoral specific intertemporal substitution effects.

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[20] Assuming that the substitution effect dominates the income effect. This assumption is not implausible for the overlapping generations model with many life periods; see Summers (1981) on this issue.

The EL-HK model

To facilitate the theoretical derivation of the intersectoral reallocation, let's rewrite physical saving for human capital production s_h as $x\left(\tilde{w}h\left(1-z\right)\right)$ where x is now the marginal propensity to save for human capital accumulation. The propensity x_{t+1} is restricted to be greater or equal to zero. A value greater than one indicates that part of human capital investment is financed from interest income or borrowing. The optimization of the intertemporal income problem consists to maximize:

$$\tilde{w}h_t\left(1-z_t\right)\left(1-x_t\right)+\frac{\tilde{w}h_{t+1}\left(1-z_{t+1}\right)\left(1-x_{t+1}\right)}{1+\tilde{r}}+\frac{\tilde{w}h_{t+2}\left(1-z_{t+2}\right)\left(1-x_{t+2}\right)}{\left(1+\tilde{r}\right)^2}$$

subject to a human capital law motion that now takes this form:

$$h_{t+1+i}=\frac{h_{t+i}}{1+\delta}+h_{t+i}^{1-a}\left(x_{t+i}\tilde{w}h_{t+i}\left(1-z_{t+i}\right)\right)^a z_{t+i}^{\gamma} \quad .$$

The first-order conditions lead to the following solution for x and z:

$$z_t=\left[\frac{\gamma^{1-a}\alpha^a\tilde{w}^a}{1+\tilde{r}}\right]^{1/1-a-\gamma} \quad ; \quad z_{t+1}=z_{t+2}=0 \quad , \tag{3.3.2}$$

$$x_t=\frac{\alpha}{\gamma}\frac{z_t}{\left(1-z_t\right)} ; \quad x_{t+1}=x_{t+2}=0 \quad . \tag{3.3.3}$$

Note that x can be easily superior to 1 if z is big enough. Moreover, $z=0\Rightarrow x=0$ and $z=1\Rightarrow x=\infty$, but the latter case, however, is certainly not a general equilibrium solution. Finally, x is monotonously related to z, so the focus can be exclusively on the latter control variable. Unlike the previous case, the after-tax wage rate is now present in the solution for z. Indeed, any decline in after-tax wage income reduces time and savings allocations to the production of human capital, other things being equal. Intersectoral reallocation of the kind previously discussed should be of second order importance. Lower human and physical capital accumulation are expected. This result is related to the symmetric case in section 3.2. Again, the endogenization of leisure complicates the analysis. Nevertheless, general intertemporal effects should dominate intersectoral and sector specific intertemporal effects.

3.4. The specification of the leisure activity

Since human capital accumulation is an engine of growth, the interaction between educational time and leisure may be meaningful for the long-run growth results. This is why this paper considers three different specifications for leisure activity: exogenous

(EL), raw-time (RT), and quality time (QT) leisure. To show the extent to which the leisure specification affects educational time, a simple example will be useful.

Consider again an education sector technology requiring only human capital as input (H models). As discussed above, the cost of investment in human capital is then forgone wages. Income taxation reduces net-of-tax wages, thus reduces benefits and costs of investing in human capital by the same proportion, leaving intact the return to the investment in human capital. However, income taxation reduces the net rate of return on physical capital, which makes human capital a more attractive investment. Through inter-sectoral reallocation, investment in physical capital declines but investment in human capital increases.[21] With an inelastic leisure activity (EL), the story stops here. However, with leisure activity specified as "raw time" (RT), the final effect on investment in human capital may differ. Since the net wage declines after a rise in income tax, individuals substitute leisure for consumption and hours worked are reduced -unless the labour supply schedule bends backward. The decline in time spent working decreases the utilization rate of human capital and therefore the return to human capital investment.[22] Consequently, time allocated to education may decline.

The quality time (QT) specification is a case between the first two. Utility from the leisure activity is now the product of leisure time and human capital stock. Since human capital has a non-pecuniary value, a greater leisure time allocation raises the marginal utility of the human capital stock. Compared with the RT specification, the utilization rate of human capital is not necessarily reduced with time reallocated from working to leisure activity. Therefore, the total rate of return to human capital investment is less affected by a decline in working time.

4. CALIBRATION

Since this paper analyzes the effects of marginal taxes on growth, the calibration procedure requires the replication of a benchmark balanced growth path. A four-step procedure is used to generate the benchmark balanced growth path, in which all stock and flow variables augment at the same rate.

The first step consists of calibrating the parameters of the final goods production function. To underline the consequences of life-cycle issues and of the various specifications for human capital production and utility functions, all six models are calibrated within the same macroeconomic environment. That is, all models are characterized with the same physical capital-output ratio, physical capital-effective labour ratio, and total factor productivity parameter. Therefore, the six economies are growing at a common rate on the benchmark balanced growth path and are observationally

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[21] Heckman (1976) and Boskin (1977) were the first to demonstrate this.

[22] See Lucas (1990) for further discussion of this.

equivalent from a supply-side perspective.[23] The factor prices that are calibrated with the first-order conditions of the firm's problem are thus common to all models.

The second and third steps serve to generate the life-cycle and cross-generational profiles. It is important to distinguish between the two profiles when growth is positive. For instance, in a two-period overlapping generations model with only consumption as argument in a CES utility function, the life-cycle consumption profile of the representative generation must satisfy the following first-order condition:

$$c_t^{old} = \left[\frac{1+r}{1+\rho}\right]^{\sigma} c_{t-1}^{young} \quad . \tag{4.1}$$

For a generation born at *t-1*, future consumption is greater than current consumption when the interest rate r is higher than the rate of time preference ρ. How much greater depends on the intertemporal elasticity of substitution σ. Equation (4.1) then serves to establish the life-cycle profile.

In a stationary state with zero growth, new generations are confronted with prices and budget constraints identical to their ancestors, and because they also have identical preferences, they repeat their ancestors' choices period after period. Therefore, one can substitute in equation (4.1) the consumption of the young generation born at time *t*, for consumption of the young generation born at time *t-1*:

$$c_t^{old} = \left[\frac{1+r}{1+\rho}\right]^{\sigma} c_t^{young} \quad . \tag{4.2}$$

Equation (4.2) serves to establish the cross-generational consumption profile. This is consistent with the life-cycle profile generated with equation (4.1). In a zero growth case, calibration procedures based on life-cycle or cross-generational data lead to identical calibrating parameters (ρ or σ).

With a positive per capita growth rate v on the balanced growth path, the cross-generational consumption profile needs to satisfy the following relationship:

$$c_t^{young} = (1+v)c_{t-1}^{young} \tag{4.3}$$

The young people at period *t* consume $1+v$ times what the young were consuming at period *t-1*. Substituting (4.3) into (4.1) produces:

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[23] A similar stategy was taken by Stokey and Rebelo (1995) in their attempt to explain the variety of tax effects obtained by different authors under a Ramsey framework.

$$c_t^{old} = \left[\frac{1+r}{1+\rho}\right]^{\sigma} (1+\nu) c_t^{young} \qquad (4.4)$$

A calibration procedure based on cross-generational consumption data which neglects the growth factor would generate incorrect life-cycle profiles and preference parameters. The effects of tax shocks on aggregate outcome would thus be biased.

The second-step calibration procedure assumes the availability of life-cycle profiles data. The preferences and human capital production parameters are calibrated using the representative generation first-order conditions and the factor prices that were generated in the first step.

Given the life-cycle profiles, the third step generates the cross-generational profiles and calibrates the parameter π. The benchmark stationary growth rate ν was chosen to be 3 percent. The parameter π is then easily calibrated using equation (2.9). Cross-generational profiles of all choice variables are generated by discounting the life-cycle profiles. For instance, the cross-generational consumption profile for the first period on the balanced growth path, is derived simply by discounting the consumption life-cycle profile of the representative generation with the growth rate as discount factor. Denoting by $\bar{c}^g$ the level of consumption at age g obtained with the calibration of the micro-profile and by c^g the consumption level of generation g in the first period of the benchmark balanced growth path, the discounting operation takes the following form:

$$c^g = \frac{\bar{c}^g}{(1+\nu)^{g-1}}, \quad g = 1,\ldots,G. \qquad (4.4)$$

This operation is repeated for all flow and stock variables of the representative generation problem.

The final step in the calibration procedure is a control subroutine. It consists of verifying whether or not the numerical model, with the calibrating parameters and stock levels, generates a dynamic equilibrium that corresponds to a balanced growth path.

Table 1 summarizes the calibrating parameters and the levels of the main variables for the six models considered. All models are calibrated with an equal tax-rate for wage and interest incomes in order to insulate further the life-cycle effects from other effects. A benchmark 15 percent rate is used, as in King *et al.* (1988). The common physical capital-effective labour ratio is 0.318, which leads to a human capital/physical capital ratio ranging from 5 to 6.6. These ratios are in the range of previous analyses. Davies and Whalley (1991) use a ratio of 3, but Jorgenson (1994) reports a ratio of 10 in value terms. Assuming that each period corresponds to 15 years, all models are calibrated with an annual rate of time preference equal to 2.4 percent, an annual human capital depreciation rate equal to 1.5 percent, and an annual intertemporal elasticity of substitution ranging from 0.157 to 0.368. The human capital sector technology is

assumed to be time intensive ($\gamma>\alpha$) in all models. On the benchmark balanced growth path, investment in human capital occurs mostly during the first period of life. Savings for physical capital are at their highest during the second period of life, which is consistent with the discussion in section 3.

5. RESULTS

Although observationally equivalent on the benchmark balanced growth path, the models differ in their responses to tax shocks because of the different specifications used for the leisure activity and the education sector. This section compares the growth effects of an unexpected rise in the marginal income flat-tax rate from 15 to 25 percent. The next subsection investigates the long-run effects. An accounting exercise then follows in order to sort out the life-cycle effects from the overall effects. The transitory path analysis closes this section.

5.1. Long-run growth and level effects

Table 2 reports that with the rise in the marginal income tax rate, long-run growth declines significantly in four models, stays almost constant in one model, and significantly increases in the other model. Negative growth effects are important (over 30 percentage points) in all three HK models, but positive effects on the accumulation of human capital are strong enough in models EL-H and QT-H to reduce significantly the negative intertemporal effect on savings, and more than offset it in the EL-H model.

The EL-H model has been referred to in the literature as the Uzawa-Lucas specification, and has been studied extensively under a Ramsey framework. In that framework, the Uzawa-Lucas specification implies that the (net of taxes) return on investment is determined by the technological parameters of the human capital production function. This return then determines the balanced growth rate.[24] A rise in the income tax rate modifies the ratio of factors used for production (or before-tax rate of returns), but does not affect the long-run growth rate (see Milesi-Ferretti and Roubini 1994). In spite of the Uzawa-Lucas specification, the EL-H model has a non-zero effect of income taxes on growth due to the presence of life-cycle effects.

Two features resulting from the Uzawa-Lucas specification in a Ramsey framework carry to the OLG framework: first, human capital remains the engine of growth; second, wage taxation remains neutral in the long run (see Lucas 1990, for the Ramsey framework case), as can be seen in the solution for educational time z in equation (3.3.1). However, there is an important divergence between the two frameworks, namely the return from human capital investment is not determined by technological parameters in an OLG model, as is the case in a Ramsey model. In an OLG model, a lower net of tax

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[24] See Lucas (1988) section 4, neglecting the external effects. See also Rebelo (1991) and Stokey and Rebelo (1995)

physical return (interest rate) raises the present value of benefits from investing in human capital. This leads to increased efforts towards human capital formation until its return equal again the net-of-tax interest rate. While young generations invest more in human capital, middle-aged generations reduce their (physical) savings because substitution effects dominate income effects. The relative decline in the physical/human capital stock ratio that follows, in turn raises the before-tax interest rate. This offsets somewhat, but not entirely, the effect of the income tax rise in the determination of the after-tax interest rate. The less than total offset, which ensure a positive long-run effect on growth, is due to a differential time horizon between the agents. Young generations, by definition, have a longer horizon plan. Hence, the positive impulse on human capital investment from young generations is stronger than the negative impulse on savings from the middle-aged. Therefore, the net of tax interest rate remains lower on the new balanced growth path. The positive growth result in the EL-H model is an example of how the presence of age-heterogeneous agents and life-cycle effects may affect the results of tax shocks on growth.[25] Note that a positive population growth rate would increase the proportion of young generations with respect to total population and thus rise human capital investments in the economy.[26]

Another example of life-cycle effects is the reported greater decline in the savings rate by the young, relative to middle-aged generations. This result can be explained by a consumption smoothing argument. Consider the intertemporal budget constraint of a representative generation on a balanced growth path:

$$c_t + \frac{c_{t+1}}{1+r(1-\tau)} + \frac{c_{t+2}}{\left(1+r(1-\tau)\right)^2} = (1-\tau)wl_t^e + \frac{(1-\tau)wl_{t+1}^e}{1+r(1-\tau)} + \frac{(1-\tau)wl_{t+2}^e}{\left(1+r(1-\tau)\right)^2},$$

where l^e is effective labour supply. With everything else constant, an increase in the comprehensive income tax rate has a greater effect on current income than on human wealth (equal to the right-hand side of the equation). Because the representative generation smooths its consumption path, first period savings for physical capital, which are the residuals of first period income $(1-\tau)wl_t^e$ minus first period consumption c_t, are reduced considerably more than are second period savings.

Other results reported in Table 2 show the role of technology and preference specifications on the absolute magnitude of the effects on growth. First, the tax shock effects on growth are relatively smaller with the H models than with the HK models.

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[25] In a two OLG economy, the life-cycle effects would be less prononced, and the effect on growth would be ambiguous (see Liu's (1994) Proposition 5).

[26] Table 2 reports that the QT-H model also experiences an increase in the effort of the young towards human capital acquisition. Therefore, a positive population growth rate could transform the negative long-run growth result into a positive one.

These simulation outcomes are consistent with the analysis of Stokey and Rebelo (1995) and in section 3.2: tax policy, when the education sector technology has only human capital and educational time as inputs, has a weak influence on long-run growth. Second, by using the reported results, it is a straightforward matter to calculate the percentage change of the rate of accumulation of both capitals with respect to the percentage change of the capital ratio. In terms of Figure 1, growth effects (changes along the vertical axis) are greater than level effects (changes along the horizontal axis) in the HK models, whereas the opposite relationship holds for the H models. This is again consistent with the technology specifications discussion in section 3.2: asymmetric technologies generate stronger level than growth effects.

Although the six models generate a variety of results, some qualitative changes are common among them. This is the case, for instance, for the decline of the physical-human capital ratio (K/H). This decline is a weighted average of the changes in educational time and savings propensities. Results reported in Table 2 show that generations' savings behaviour are more sensitive to marginal tax changes than is educational time behaviour.

The relatively smaller absolute change (in percentage) of generations' educational time in all models stems from three of the aforementioned particular dimensions of human capital. First, leisure time is bounded below at an allocation greater than zero by the Inada condition on the utility function. Also, the budget constraint requires a positive labour supply during at least one period over the lifetime. Therefore, educational time is bounded above at an allocation strictly below one. Educational time is also bounded below at an allocation greater than zero by the Inada condition on the education production function, which ensures that the marginal productivity of educational time tends to infinity at very small levels of investment. Notice that savings are bounded above by intertemporal budget constraints, and below by the Inada condition for consumption in the utility function. As households are return-takers for physical capital, savings can thus be negative. Second, inputs used in the human capital production technology are taxed only indirectly as a consequence of the non-substitutability of human capital with final goods. Third, the non-marketability of human capital implies that the de-accumulation of this stock of wealth (at a rate above the depreciation rate) is not permitted. Given these particular properties of human capital, there is less scope for absolute percentage change in educational time than for savings.

Different directions in educational time responses across generations are observed for all four RT and QT models. In three cases out of four, the middle-aged generation increases its educational time while the young generation reduces it.[27] The opposite relationship occurs in the other model. However, the savings responses of the two

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[27] Of course, if during the first two periods of life savings decline, dissavings during third period of life (savings) also decline (augment). The positive numbers reported in Table 2 for percentage changes in the savings rate of the old generation, thus indicate a decline in dissavings.

investor generations go in the same direction. These directional age-heterogeneous responses on educational time, in comparison to the directional age-homogeneous responses on physical capital accumulation, lessen the impact of tax changes on human capital accumulation at the aggregate level, relative to the impact on physical capital accumulation. The combination of concordant savings responses with discordant educational ones reflect the divergent life-cycle properties of human and physical capital. Because physical capital can be bought and sold any time, the life-cycle first-order conditions are separable into two-period intertemporal conditions. In other words, savings decisions when middle-aged are independent of the savings decisions made when young. Since tax changes affect those separable conditions similarly, the young and middle-aged generations react similarly. This means that after a tax shock, life-cycle savings profiles shift. However, because human capital is embodied, the educational time decisions at the middle-aged period is dependent on the school time decision taken when young. Therefore, the after-tax revision of the whole educational time profile cannot be separated into repeated two-period trade-offs. The sign of the change may differ across age and the revised (after-tax) profile may cross the before-tax profile.

The general decline in the physical-human capital ratio leads to similar decreases in physical-effective labour (K/L_e) and output-effective labour ratios (Y/L_e). For both human capital production specifications, the decrease in physical capital-effective labour ratio is larger than the decrease in physical-human capital ratio in EL models, while the opposite relationship holds for RT and QT models. In those two types of models, the private agents dedicate more time to leisure after a rise in income taxes (see again Table 2), given the reduced opportunity cost. Thus, effective labour supply decreases further, for a given decline in physical capital. These results are consistent with Lucas' (1990) argument, discussed in section 3.4, claiming that a lower capacity utilization rate of human capital leads to a lower return for human capital investment. Correspondingly, a lower return generates less human capital accumulation and thus a higher physical-human capital ratio. In summary, the Table 2 results show that in an OLG endogenous growth model, the intersectoral reallocation effects are important and the trade-off between investment in physical capital and human capital is more likely to discriminate against physical capital accumulation after a marginal income tax increase.[28]

It is well known that changes in the growth rate causes compounding effects on long-run output, and hence on welfare. Table 3 reports the extent of these effects in comparing the final goods output and welfare of a generation born 15 periods after the tax shock with those of a generation born at the same time, but in an economy where no change in

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[28] This result clashes with the one obtained by Nerlove *et al.* (1993). They show that a comprehensive tax to labour and capital incomes discriminates against investments in human capital relative to investments in physical capital. But their set-up does not have an endogenous growth feature and their human capital production function is intensive in physical capital.

tax policy has occurred.[29] Note again that the percentage changes reported are more substantial in HK than in H models. The magnitude of these changes depend on the transitional path and the speed of convergence to the new balanced growth path. Figures 3 and 4 show the transitory path of the final goods output for H and HK models. Figure 3 shows that in all H models, there is at some points on the transitory path a higher growth rate than the benchmark three percent. Such increases in the growth rate are rare for HK models, because general intertemporal substitution effects dominate. The temporary rise in the growth rate for RT-H and QT-H models, in particular, explains the relatively lower change in income and welfare of the living generations after 15 periods.

The sensitivity analysis

Table 4 reports some sensitivity analysis. Three experiments were simulated. In the first two experiments, the same tax increase (from 15% to 25%) was repeated but with lower share parameters for ε and $\alpha+\gamma$. A third one consists in retaking original predetermined preferences and production functions parameters' values but calibrating the benchmark balanced growth path with 25 percent tax rates, and simulating an unexpected 10 percent tax rise. The above analysis is robust to these experiments. Note, however, the increase in the growth rate for the QT-H model when ε=.35. As indicated in Stokey and Rebelo (1995), production factor shares matter for quantitative results.

5.2. Growth accounting incidence

Although it is not possible to identify exactly the role of every behavioural channel behind the growth results, this section assesses the relative importance of each of them through an accounting exercise.

For that purpose, a broad output, Y_B, is defined as the sum of the output in the final goods and in the education sectors. With the linear homogeneous properties of production in the two sectors, broad output is also equal to factor incomes used in both sectors:

$$Y_B = r\sum_i k_i(s_i) + w\sum_i (1-l_i)h_i(z_i, x_i) \ .$$

Private behaviour alters broad output through four channels. Sectoral revision of private investment changes the *H/K* ratio, which is reflected in the factor prices ratio channel in the broad output equation. Changing behaviour toward leisure time is reflected in the productive utilization rate of human capital.[30] Changing behaviour in savings and

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[29] This comparison avoids the need to define a social welfare function. This approach is certainly unsatisfactory for welfare analysis, but is still useful for positive analysis.

[30] The productive utilization rate for human capital includes the portions used in the final goods sector and in the education sector, but excludes the portion used in the leisure activity (for QT models).

educational time entails changes in physical and human capital stocks. To assess the role of each channel in determining long-run growth, long-run broad output is compared to a series of accounting output. Each accounting output is derived by substituting for each behavioural variable in the broad output equation, the value that would have been obtained if private agents were behaving, with respect to that variable, as in the benchmark case. For instance, one accounting output is obtained by replacing the simulated long-run factor prices with the benchmark equilibrium values; a second, by replacing the long-run physical and human capital stocks with those that would have been obtained with savings and educational time behaviour as in the benchmark equilibrium; a third, by replacing long-run leisure time with the benchmark allocation.

Table 5, upper part, reports the relative importance of each behavioural channel in comparing the growth rate obtained in the simulations with the corresponding accounting result. In this table, rank 1 designates the channel that generates the greatest change in the growth rate. Although the ranking must be interpreted with caution, it is worthy of the following observations: first, changes in behaviour towards productive activities (labour and educational time) are most important in modifying the growth rate in all models in which this behavioural change is applicable (RT and QT models); second, the relative importance of change in human capital is greater for HK models than for H models; third, factor price changes are relatively less important for HK models than for H models. The second and third observations are two alternative ways of stating that intersectoral effects are more important when human capital is the engine of growth (H models).

In a Ramsey framework, substitution effects change the absolute level of savings and time allocation. In an OLG framework, these substitution effects change not only the levels but also the shape of the savings and time allocation profiles. The lower part of Table 5, reports another accounting exercise which tries to sort out profile and absolute level changes. In this exercise, the accounting output is calculated by imposing the same capital and time allocation profiles obtained in the benchmark balanced growth path. The accounting growth rate is calculated by dividing the accounting output by the broad output obtained in the next-to-last period of simulation. The difference between the benchmark growth rate (3%) and the accounting growth rate gives the substitution effect, since the shape of the life-cycle profiles are similar in the two experiments.[31] The accounting growth rate, minus the simulated long-run growth rate, gives the life-cycle effect, since substitution effect is similar in the two experiments. A division of the latter over the former, gives the relative importance (in absolute terms) of the life-cycle effect with respect to the substitution effect. For each of the six models, the table reports an interval for this ratio as the life-cycle effect was calculated with two different anchors (first and last periods of life) for the life-cycle profiles. The two anchors give a lower and an upper boundary of the relative importance of the life-cycle effect over the substitution

[31] A similar life-cycle profile means, for instance, a constant middle-aged/young ratio for the accumulated stock of human capital.

effect. As can be seen in the bottom part of Table 5, the relative importance of the life-cycle effect is significant in all models. The intervals suggest a domination of the life-cycle over the substitution effect in the EL-H, RT-H, RT-HK, and QT-HK models. As expected, the smallest interval is obtained with the EL-HK model, while the largest one is obtained with the EL-H model.

5.3. Transitional path analysis

The six panels of Figure 5 depict the transitory path of the growth rate for broad output, the final goods sector, and GDP. As in Mulligan and Sala-i-Martin (1993), final goods output accounts for 75 percent of GDP, while educational output accounts for 25 percent. In all six models, the dynamic paths of the three measures of output differ mainly at period zero (when the new tax policy is introduced). Because broad output comprises the productive activity of both sectors, it is less affected by intersectoral adjustments. Broad output changes are thus weaker than final goods output changes, whereas GDP growth changes are between the former two. A notable exception is the EL-HK model, in which intersectoral adjustments are restricted by the leisure and human capital specifications. In this model, the growth rates of final goods output and GDP are greater than the 3 percent benchmark rate at period zero, because the reduction in educational time is compensated by an increase in working time. The similarity of the six dynamic paths for the three measures of output after period zero suggests that the intersectoral adjustment takes place in the short run. The fact that educational output eventually fuels into final goods output and that broad output is, by definition, less sensitive to intersectoral reallocations, suggests that broad output could be a good predictor of future GDP.

The six panels of Figure 6 depict the dynamic growth path of human and physical capital, and of effective labour supply. In all models, the physical capital accumulation rate falls sharply in response to the comprehensive tax rise, and remains mainly negative on the transition path. In contrast, the human capital accumulation rate never becomes negative on the transitory path. This accumulation rate even exceeds the benchmark three percent rate in four models. Because capital stocks are fixed at period zero, the immediate response to the new tax policy is through time allocation. In the EL-H model, a higher income tax rate implies a higher rate of return on human capital investment, therefore more effort is immediately allocated towards the education sector, and the effective labour growth rate is reduced. In EL-HK, the new tax policy has the opposite effect on human capital returns, hence effective labour increases. When leisure is endogenous (RT and QT models), the tax-hike is absorbed at period zero by important reductions in the effective labour supply. The fraction of the time endowment allocated to the labour market (not reported here) falls immediately and remains at a lower level than in the benchmark case. However, the working time response in the short run overshoots its long-run response. On the transitory path, therefore, absolute changes in the effective labour growth rate are greater than those in the human capital accumulation rate. It is worthwhile noting that, because of its characteristics, changes in human capital are less volatile than those in effective labour and physical capital. Convergence in the

accumulation rate of the three stocks occurs after six periods, in all models, with the exception of the RT-HK model.

Figure 7 depicts the transitory path of the human-physical capital ratio for the six models. In all models the pattern is the same: the ratio increases in the short run, but diminishes to a lesser degree in the medium run. There is a similarity between this pattern and the output patterns of Figure 5.

Figures 3 and 4 depict the dynamic path of the final goods growth rate for H and HK models respectively. The figures offer insights into the specification used for leisure activity. Note that the dynamic path under the QT specification is always above the path under the RT specification, when human capital is the engine of growth (H models). It is generally above when physical and human capital are the engines of growth (HK models). These relationships are due to the non-pecuniary value of the human capital stock under the QT specification. When leisure is exogenous (EL models), growth depends only on the fraction of time allocated to the production of human capital. In EL-H model, intersectoral and sector-specific intertemporal substitution effects lead to an immediate and permanent increase in the growth rate. In the EL-HK model, negative intertemporal substitution effects for both capitals lead to an immediate and permanent decline in the growth rate.

Figures 8, 9 and 10 depict the dynamic path of the final goods output growth rate for the EL, RT, and QT models, respectively. They offer insights into the role of the education sector specification. Note that the dynamic path of growth with HK models evolves under the paths generated by the H models. Moreover, the HK models generate a larger number of and more erratic cycles, especially when leisure has the raw time specification.

6. CONCLUSION

A three-period overlapping generations model was solved numerically to evaluate the effects of income taxes on growth. Life-cycle issues were emphasized by calibrating a benchmark stationary state with equal flat-rate taxes for interest income and wage income. Three different specifications for the leisure activity and two for the human capital technology were covered in this paper. It has been shown that life-cycle considerations enlarge the scope for intersectoral reallocation and sector-specific intertemporal substitution, and that responses to tax shocks are less dependent on preference and technology parameters. Moreover, it has been demonstrated how the specification adopted for human capital production and for the leisure activity alter the growth results. In all six models, an unexpected rise in the marginal income tax rate from 15% to 25% was performed. The outcomes are often different than in a Ramsey framework. The main results of the simulations are:

(1) A rise in income taxes may *stimulate* growth. The model components necessary to generate that kind of counter-intuitive result are: asymmetric technologies, a human capital technology intensive in time, and a low labour supply elasticity. Asymmetric

technologies encourage intersectoral reallocation and sector-specific intertemporal substitution effects. Intensity in time of the human capital technology implies, other things being equal, neutrality of the wage tax, but a positive effect of the interest income tax on human capital investment. The latter effect stems from the increase in the present value of the stream of future labour income. A low labour supply elasticity impedes significant change in the utilization rate of human capital. In an Uzawa-Lucas specification, for instance, the rise in income taxes stimulates growth with certainty, because human capital is the real engine of growth in this model.

(2) A model with asymmetric technologies between the two sectors leads to weaker absolute effects on growth than a model with symmetric technologies.

(3) A model with asymmetric (symmetric) technologies leads to relatively stronger (weaker) level effects than growth effects in the long run.

(4) In all six models, human capital accumulation is less sensitive to the marginal tax rate than is physical capital accumulation. Hence, in the long run, a comprehensive rise in income taxes discriminates against investment in physical capital relative to investment in human capital. This result stems from the aforementioned distinctions between human capital and physical capital. When the labour supply is endogenous, the effect of this bias on the physical capital-effective labour ratio is reduced by the fact that agents allocate less time to work.

(5) In models with a human capital technology independent of physical capital (H models), a comprehensive rise in income tax generates a short-run increase in the growth rate, although the long-run effects are usually negative. This temporary rise in the growth rate lessens the negative medium-run effect on the level of income and on welfare.

(6) Changes in behaviour towards productive activities (working and educational time) are the most important channels modifying the growth rate in models in which this behavioural change is applicable (RT and QT models).

(7) Changes in individuals' life-cycle profiles are as important as changes in absolute levels of savings and time allocation.

(8) Broad output, which is a measure of total production in an economy, is a good predictor of future GDP.

(9) On the transitory path, the human capital stock evolves in a less erratic way than do effective labour and physical capital. Moreover, the short-run response of working time to a tax hike, overshoots its long-run response.

(10) The quality time leisure specification adds a significant incentive for investment in human capital. The growth effects under that specification are improved in comparison with the raw time specification, and differ from the exogenous leisure specification.

Because previous analyses of taxation in a two-sector endogenous growth model have been carried out in a Ramsey framework, it is pertinent at this point to compare the ten aforementioned results with those found in that literature. A long-run increase in the growth rate after a rise in comprehensive income tax rates (Result 1) can never happen in a Ramsey framework, while short-run increases (Result 5) have never been reported in numerical Ramsey type of analyses. The equivalence under the Ramsey framework between quality time and exogenous leisure specifications (see Proposition 3 in Milesi-Ferretti and Roubini 1994) no longer applies (Result 10). Results (3), (6), (7), and (8) were not discussed in previous analyses. Result (2) is, however, similar to results reported in Ramsey type analyses. Finally, Result (4) clashes with the analysis of Nerlove *et al.* (1990, 1993), who also use an OLG framework.

For those who believe in the lack of empirical evidence for any substantial tax-distortion effect on the long-run growth rate (as Stokey and Rebelo, 1995), the QT-H model may be the most preferred since it generates mild long-run growth effects. However, the short-run effects are important in the QT-H model. Finally, it has to be emphasized that given the very abstract nature of the models used in this paper, the results must be regarded cautiously and as merely the first stage in indentifying growth effects of income taxes. Actual economies are very complex, comprising many structural details, linkages and engines of growth that the simulation models do not presently capture. Further research and modelisation effort is thus planned in this regard.

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Appendix I

Proposition: A linear social heritage mechanism, combined with linear homogeneous (individual) human capital technologies in the accumulating inputs, are sufficient to make aggregate human capital technology linearly homogeneous with respect to the accumulating inputs.

Proof: Assume that human capital production function takes the following form:

$$h_{t+1}^{g+1} = B\left(h_t^g, s_{h,t}^g, z_t^g; \delta\right) \tag{A1.1}$$

where all variables are defined as in the main body of the text. The production function B is assumed linear in h and s_h. Since the production function is independent of time, we can derive the following aggregation condition for the economy:

$$\begin{aligned} H_{t+1} - H_t &= \sum_{g=1}^{G} h_{t+1}^g - \sum_{g=1}^{G} h_t^g = \sum_{g=2}^{G} h_{t+1}^g - \sum_{g=1}^{G-1} h_t^g + h_{t+1}^1 - h_t^G \\ &= \sum_{g=2}^{G} B\left(h_t^g, s_{h,t}^g; z_t^g\right) + \pi \sum_{g=1}^{G} h_t^g - h_t^G \end{aligned} \tag{A1.2}$$

The last line in A1.2 is derived using A1.1 and (2.9). The first term of this last line is a sum of linear homogeneous functions in h and s_h. The last two are linear in h. Obviously, the aggregate accumulation technology is also linearly homogeneous in h and s_h. So the marginal product of human capital at the aggregate level does not decline with respect to its stock, even if it does from the individual's perspective. The technologies described in the model by equations (2.1), (2.7), (2.8) and (2.9) permit long-run growth, since the marginal products of both capital goods are bounded away from zero at the aggregate level.

Appendix II

A Geometric Exposition of a Two-Sector Endogenous Growth Model

This appendix explains, through the use of an example, the graphical exposition in Figure 1 and Figure 2 used for the two-sector model. Since the two figures are used to discuss intersectoral allocations and technology specifications, the overlapping generations structure is left out in the following description. The structural equations of the example are as follows:

First, the utility function is assumed to be intertemporal iso-elastic with consumption and leisure activity as arguments:

$$U = \int_0^{\infty} e^{-\rho t} u(C_t, L_t) dt \qquad \text{(A2.1)}$$

Second, the resource constraint of the economy requires that investment in physical capital be equal to output net of depreciation of physical capital, minus aggregate consumption:

$$dK = A(qK, uH) - \delta K - C \qquad \text{(A2.2)}$$

Net investment in the education sector equals human capital output, minus depreciation of the accumulated stock:

$$dH = B(xK, zH) - \delta H \qquad \text{(A2.3)}$$

Both A and B functions are linearly homogeneous in K and H. Leisure activity depends on leisure time l and human capital H:

$$L = lH \qquad \text{(A2.4)}$$

The variables l, u, and z are the fractions of the time endowment allocated to leisure, labor supply, and human capital production, respectively:

$$l + u + z = 1 \ . \qquad \text{(A2.5)}$$

The variables q and x are the fractions of physical capital allocated to physical and human capital productions, respectively:

$$q + x = 1 \ . \qquad \text{(A2.6)}$$

The balanced growth path of this economy is characterized by constant and equal growth rates for K, H, C, and L. All fractions l, u, z, q, and x remain constants on the stationary path. Exploiting the linear homogeneity of the production function, one can write equations (A2.2) and (A2.3) as:

$$\frac{dK}{K} = A\left(\frac{qK}{uH}\right) - \delta - \frac{C}{K} \qquad \text{(A2.7)}$$

$$\frac{dH}{H} = B\left(\frac{xK}{zH}\right) - \delta \qquad \text{(A2.8)}$$

with first derivatives $A'<0$ and with $B'>0$. The dH/H schedule is obviously upward sloping with respect to K/H. Using the first-order conditions of the representative agent problem and the two transversality conditions (not written here), it can be shown (see Milesi-Ferretti and Roubini, 1994) that C/K depends on the same input ratio as function A:

$$\frac{C}{K} = F\left(\frac{vK}{uH}\right), \qquad \text{(A2.9)}$$

with first derivative $F'<0$. Under plausible parameter values, the marginal productivity effect is larger than the marginal utility effect, after an increase in the ratio K/H. So $A'>F'$ and the dK/K schedule is declining in K/H.

Appendix III

Trade-offs between Investments in Human and Physical Capital

This appendix shows the trade-off distortions between physical and human capital accumulation that are created by the presence of flat-rate income taxes.

Assume an investor considering an investment in human capital and in physical capital. Two means of financing for the investment in human capital are examined. In the first, investment in human capital is financed by forgoing consumption, whereas in the second, investment is financed by forgoing time in the labour market (wage income). Let w be the relative wage rate per unit of time. By investing Z (Zw) units of forgone consumption (wage income) in human capital, the investor increases his earning capacity in the future through the function $B(Z)$ where $B(0)=0$, $B'>0$, $B''<0$. Alternatively, if she invests those Z (Zw) units in physical capital, she gains a return rZ (rZw) at each future period where the rate r is determined by the physical capital market.

Suppose that the investor's preferences depend only on consumption (as in the EL models). The investment allocation between human and physical capital is thus derived from the maximization of intertemporal income. If she lives indefinitely, the net present value of investing an additional unit in human capital is $wB'(1)/r$ for both types of financial means. Alternatively, the net present value of investing one unit of forgone consumption (wage income) in physical capital is 1 (w). A no-arbitrage condition between investment in physical and human capital, requires an equality at the margin between the returns from both forms of investment. Thus, the following no-arbitrage condition applies for the first case (when the cost of human capital investment is forgone consumption):

$$wB'(Z)/r = 1 \qquad \text{(A3.1)}$$

and for the second case (when cost of investing is forgone wage income):

$$wB'(Z)/r = w \qquad \text{(A3.2)}$$

In the first case, if labor and interest incomes are both taxed at the rate τ, equation (A2.1) becomes:

$$\frac{(1-\tau)w}{(1-\tau)r}B'(Z) = 1 \qquad \text{(A3.3)}$$

and in the second case, equation (A2.2) becomes:

$$\frac{(1-\tau)w}{(1-\tau)r}B'(Z)=(1-\tau)w \qquad \text{(A3.4)}$$

Obviously, from equation (A3.3) the two income tax rates offset each other and the trade-offs between human and physical capital investments are not affected. In (A3.4), the trade-offs are distorted in favour of investment in human capital, since the *B* function is strictly concave.

However, if the investor has a finite lifetime and if the human capital asset is embodied, human capital depreciates to zero at retirement. As depreciated human capital is not tax-deductible, the income tax is levied on both principal and returns. Because the principal of physical capital can be sold in the physical market, returns to investment in physical capital are as before. The no-arbitrage conditions for a marginal investment in human capital *T+1* years before retirement are:

$$\frac{(1-\tau)w}{(1-\tau)r}B'(Z)\left[1-\left(\frac{1}{1+(1-\tau)r}\right)^{T}\right]=1 \qquad \text{(A3.5)}$$

when that marginal investment in human capital is financed through forgone consumption and

$$\frac{(1-\tau)w}{(1-\tau)r}B'(Z)\left[1-\left(\frac{1}{1+(1-\tau)r}\right)^{T}\right]=(1-\tau)w \qquad \text{(A3.6)}$$

when that marginal investment in human capital is financed through forgone wage income. Since the term in $[\cdot]$ is less than one, equal flat-rate taxes for wage and interest income introduce trade-off distortions against investment in human capital in the first case, but distortions in favour of human capital (although weaker than in A3.4) in the second case.

The above analysis is necessarily limited. In a general equilibrium Ramsey model, the real interest rate is closely tied to the *B(Z)* technology parameters when investment in human capital is financed through time allocation. So the trade-off distortions in favour of human capital implied by equation (A3.4) may be very mild. This is the case for instance in the Lucas (1990) model. In the overlapping generations framework, however, there is less connection between the real interest rate and the technologies and/or preference parameters. Therefore, the trade-off in favour of human capital in the second case could be real only for the OLG framework.

TABLE 1: CALIBRATION

First Step Calibration

Predetermined values (all models)

Y	L_e	K	ε	τ_r	τ_w
1	1	.318	.35	.15	.15

Endogenous values

A	w	$\tilde{w}$	r	$\tilde{r}$
1.493	.65	.55	1.1	.935

Second Step Calibration

Predetermined Values: common to all subcases: ρ=.429; δ=.3;

	EL-H	RT-H	QT-H	EL-HK	RT-HK	QT-HK
θ	0	.35	.35	0	.35	.35
γ	.7	.7	.7	.4	.4	.4
α	0	0	0	.3	.3	.3
β	2	2	1.5	4	3.5	3.5
ϕ	0	0	1	0	0	1

Endogenous Variables:[1]

	EL-H	RT-H	QT-H	EL-HK	RT-HK	QT-HK
z^1	.34	.239	.285	.22	.326	.346
z^2	0	.022	.029	0	.023	.043
l^1	0 (exog.)	.23	.2	0 (exog.)	.165	.1
l^2	0 (exog.)	.283	.32	0 (exog.)	.148	.204
l^3	1	.82	.795	1 (exog.)	.884	.8
x^1	n/a	n/a	n/a	.211	.48	.469
x^2	n/a	n/a	n/a	0	.02	.042
s^1	.236	.316	.386	.255	.017	.463
s^2	.451	.453	.376	.482	.713	.595
s^3	-.909	-.695	-.662	-.909	-.774	-.669
σ	2.357	2.956	2.636	2.698	5.522	4.712

Third Step Calibration: ν=.03.

	EL-H	RT-H	QT-H	EL-HK	RT-HK	QT-HK
π	.264	.275	.292	.251	.217	.204
H/K	5.32	6.62	6.8	5.08	6.41	6.41

[1] z and l are fractions of time allocated to human capital production and leisure respectively. x and s are marginal propensities to save in human and physical capital respectively.

TABLE 2: PERCENTAGE CHANGE BETWEEN STEADY STATES

	EL-H	RT-H	QT-H	EL-HK	RT-HK	QT-HK
v	18.8	-11.1	-0.01	-62.2	-34.8	-33.0
K/H	-14.2	-17.2	-17.3	-9.1	-18.5	-17.5
K/L_e	-14.8	-12.3	-11.8	-9.7	-14.0	-12.5
Y/L_e	-5.3	-4.28	-4.4	-2.3	-4.9	-4.4
z^1	4.1	-3.4	2.5	-5.9	-1.5	-1.2
z^2	0	4.5	-17.2	0	21.7	18.6
l^1	n/a	15.2	15.0	n/a	15.2	22.0
l^2	n/a	-24.7	8.1	n/a	17.6	14.7
l^3	n/a	3.2	4.3	n/a	2.1	2.25
x^1	n/a	n/a	n/a	-7.1	2.5	2.1
x^2	n/a	n/a	n/a	0	35.0	26.1
s^1	-22.9	-16.1	-15.0	-16.5	-505.9	-15.1
s^2	-2.7	-4.2	-1.6	-5.8	-4.1	-5.5
s^3	8.7	5.6	4.1	5.9	7.2	7.3

TABLE 3: CUMULATIVE PERCENTAGE CHANGE AFTER 15 PERIODS

	EL-H	RT-H	QT-H	EL-HK	RT-HK	QT-HK
Y	1.09	-10.9	-8.0	-25.2	-22.4	-18.6
U	1.4	-1.9	-0.2	-15.0	-9.2	-9.7

TABLE 4: SENSITIVITY ANALYSIS

		EL-H	RT-H	QT-H	EL-HK	RT-HK	QT-HK
ε=.25	v	12.6	-9.0	1.2	-2.7	-24.2	-16.8
	K/H	-13.7	-16.9	-23.3	-10.0	-17.2	-14.7
$\alpha+\gamma$=.5		γ=.5	γ=.3;α=.2	γ=.3;α=.2	γ=.5	γ=.3;α=.2	γ=.3;α=.2
	v	10.4	-4.0	-2.6	-31.0	-24.9	-19.3
	K/H	-13.4	-18.5	-16.7	-11.5	-17.9	-18.1
τ=.25	v	18.6	-14.3	-1.7	-79.7	-32.9	-31.7
	K/H	-18.6	-20.2	-20.0	-10.2	-22.0	-20.8

TABLE 5: GROWTH ACCOUNTING

	EL-H	RT-H	QT-H	EL-HK	RT-HK	QT-HK
	Rank	Rank	Rank	Rank	Rank	Rank
Physical Capital	1	2	2	1	2	2
Human Capital	3	4	4	2	3	3
Factor Prices	2	3	3	3	4	4
Productive Time	N/A	1	1	N/A	1	1
	Percent	Percent	Percent	Percent	Percent	Percent
Life-Cycle /Substitution	110-213	95-115	100-102	26-50	80-167	90-145

Figure 1

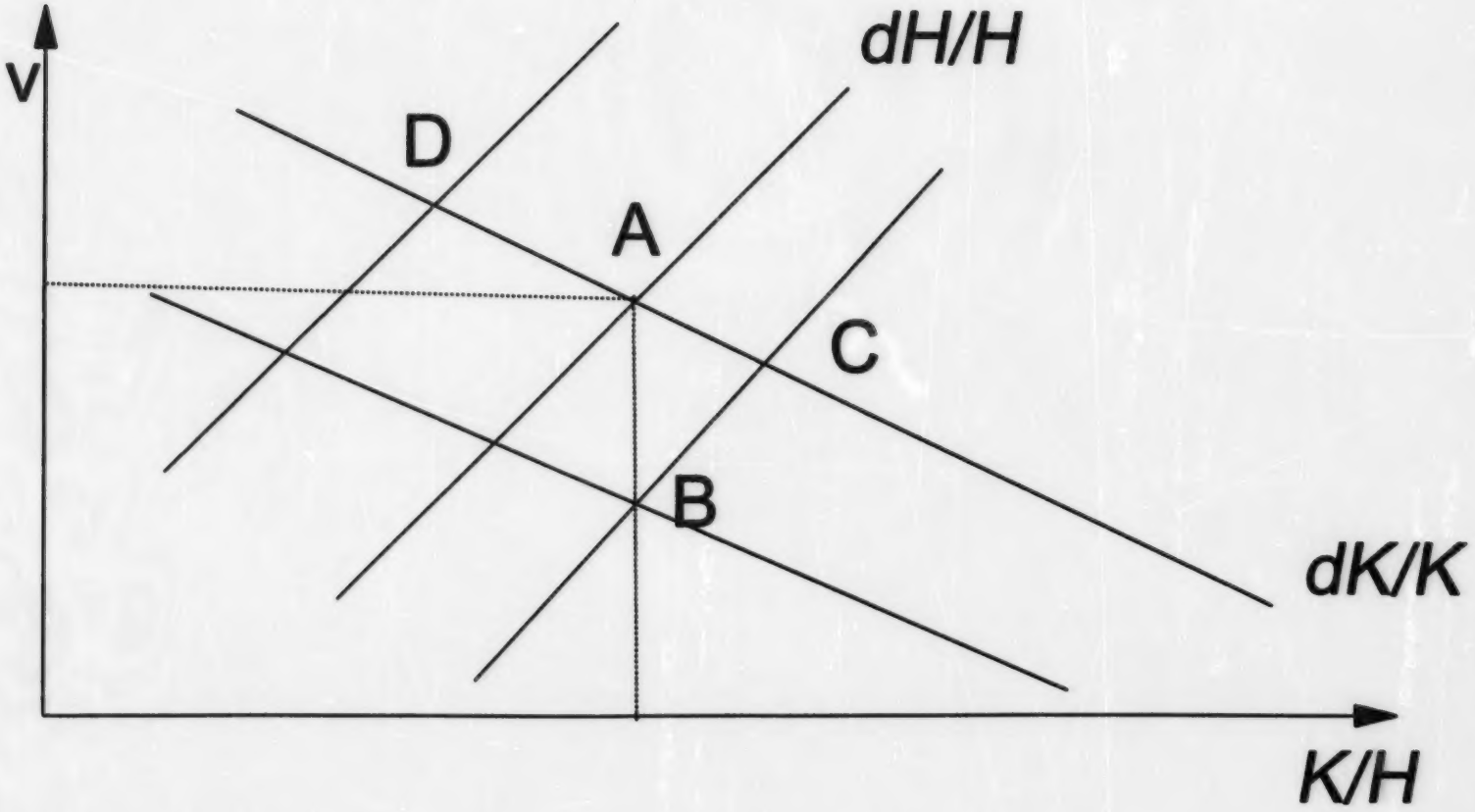
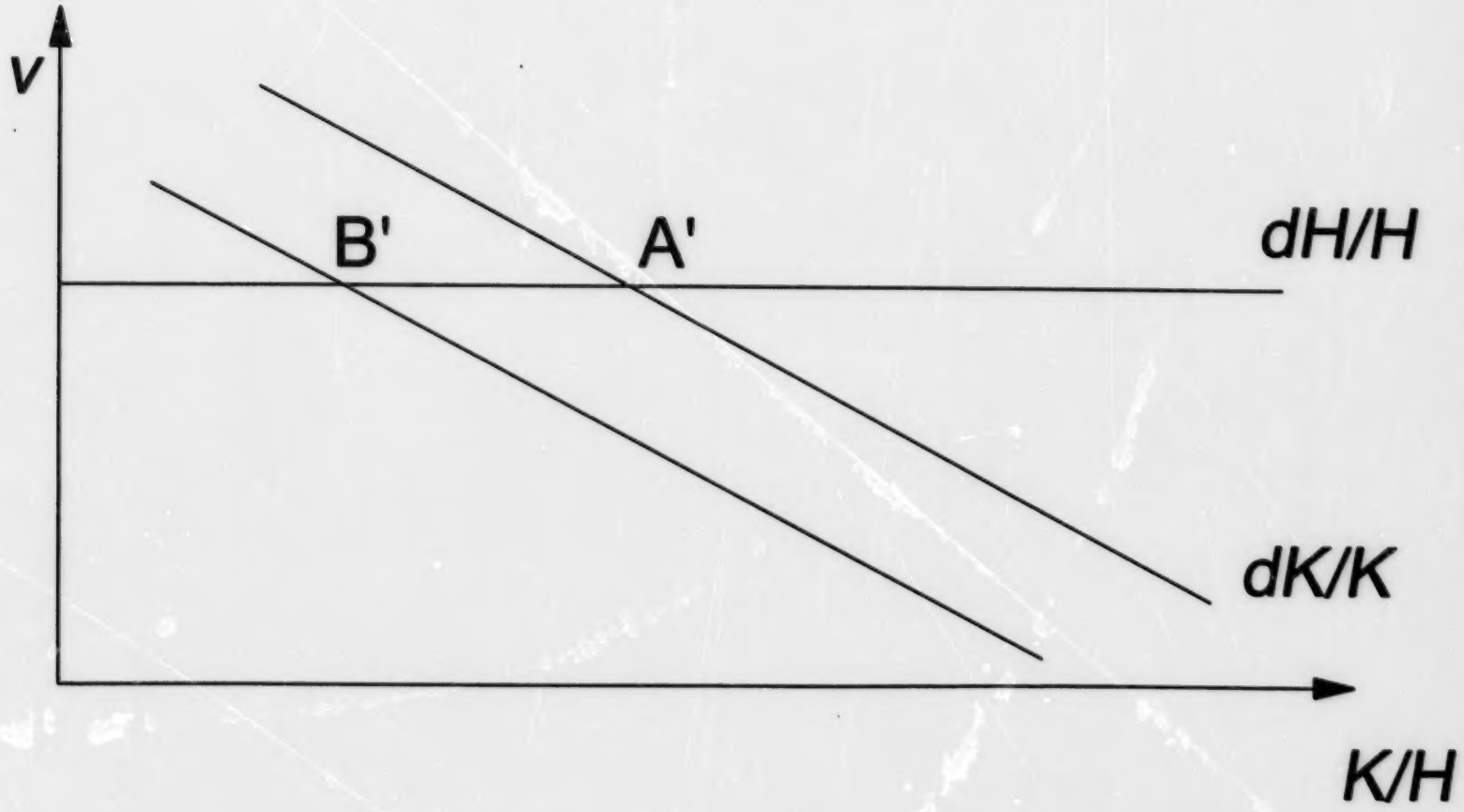


Figure 2



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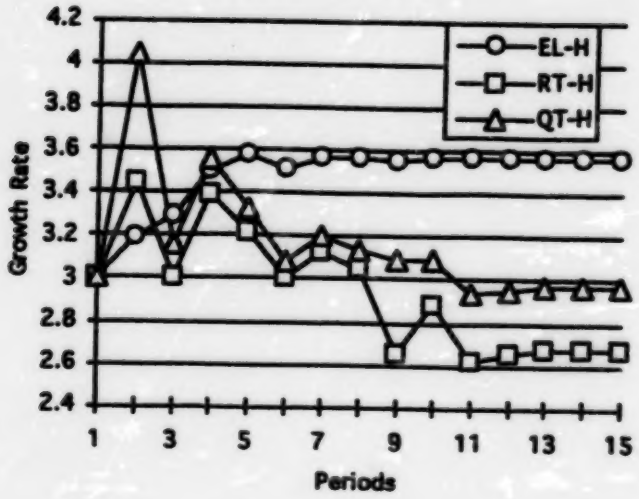

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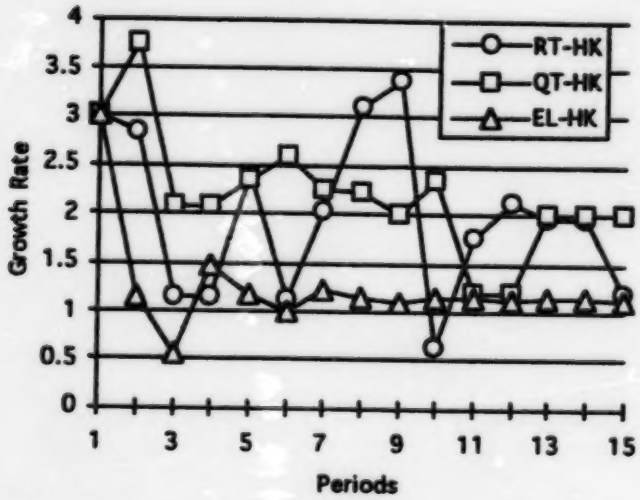

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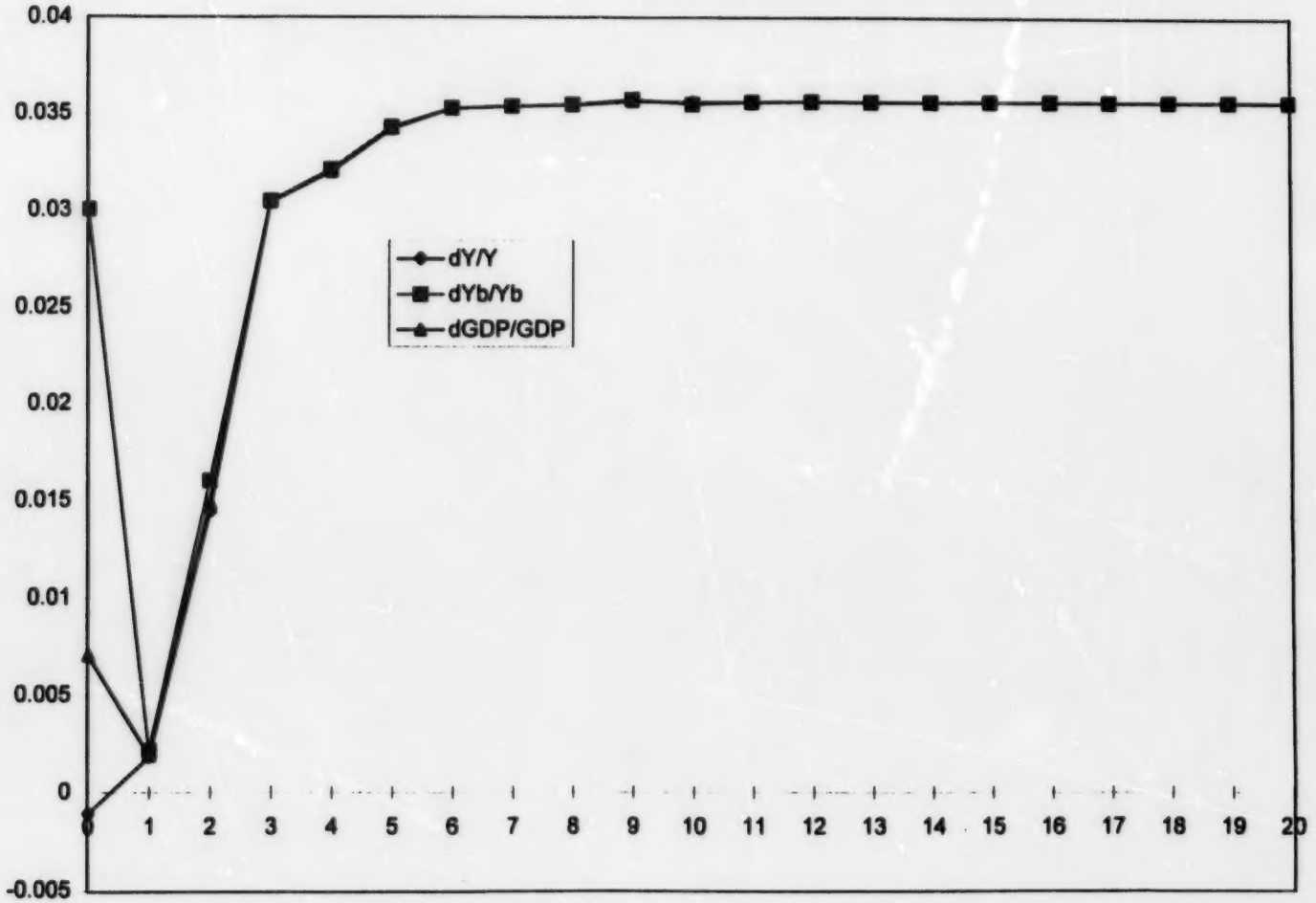
MODEL EL-H



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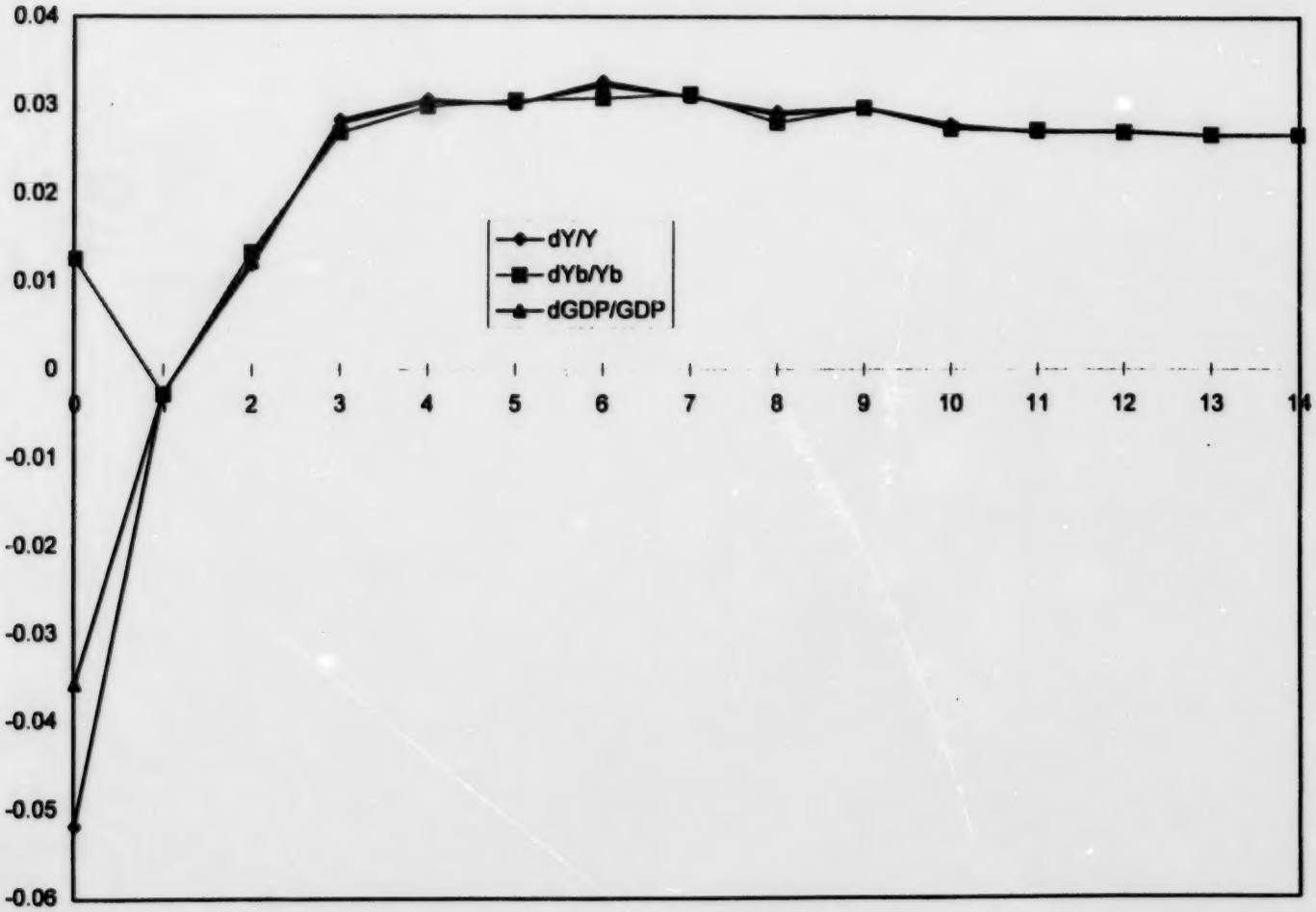
MODEL RT-H



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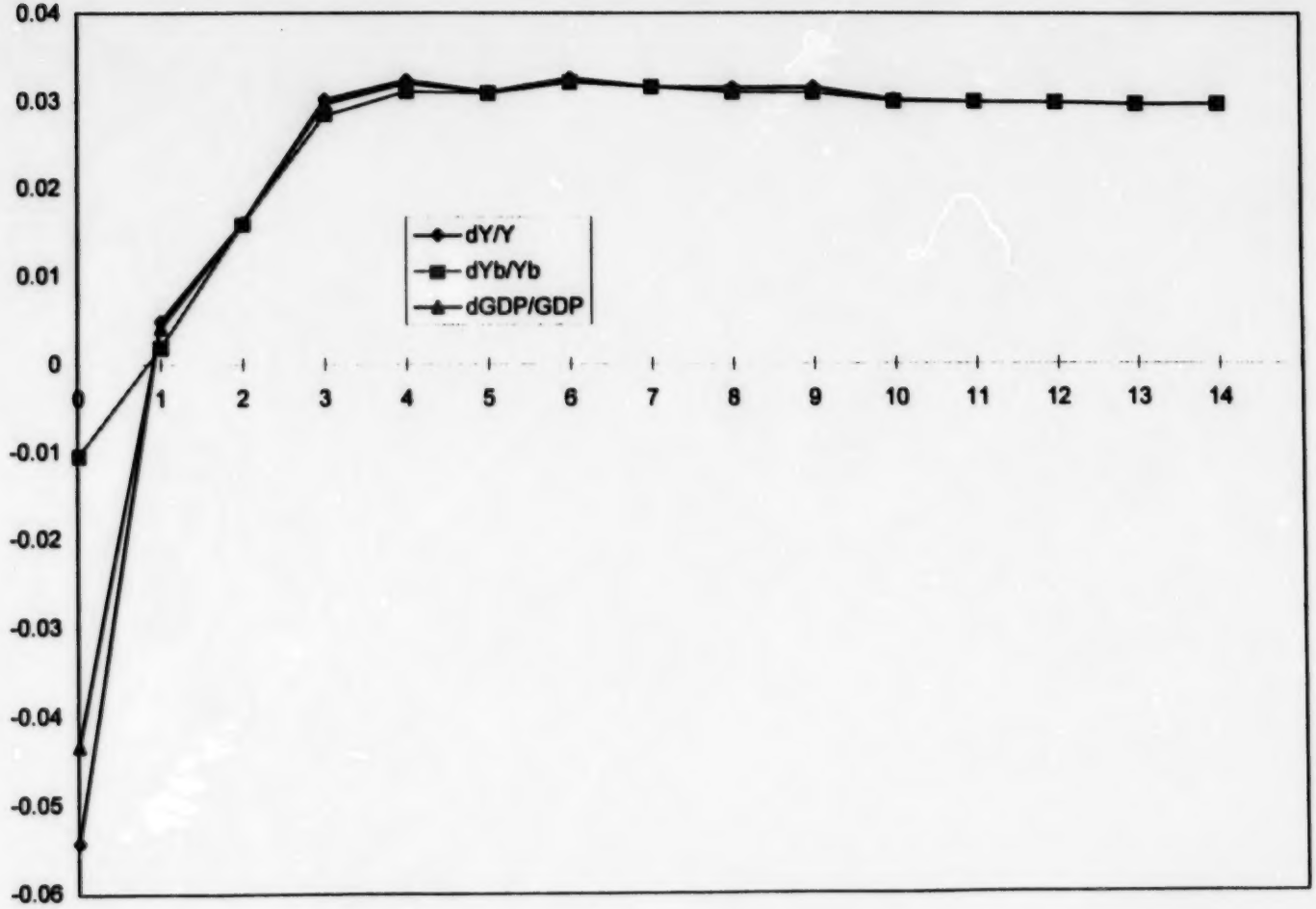
MODEL QT-H



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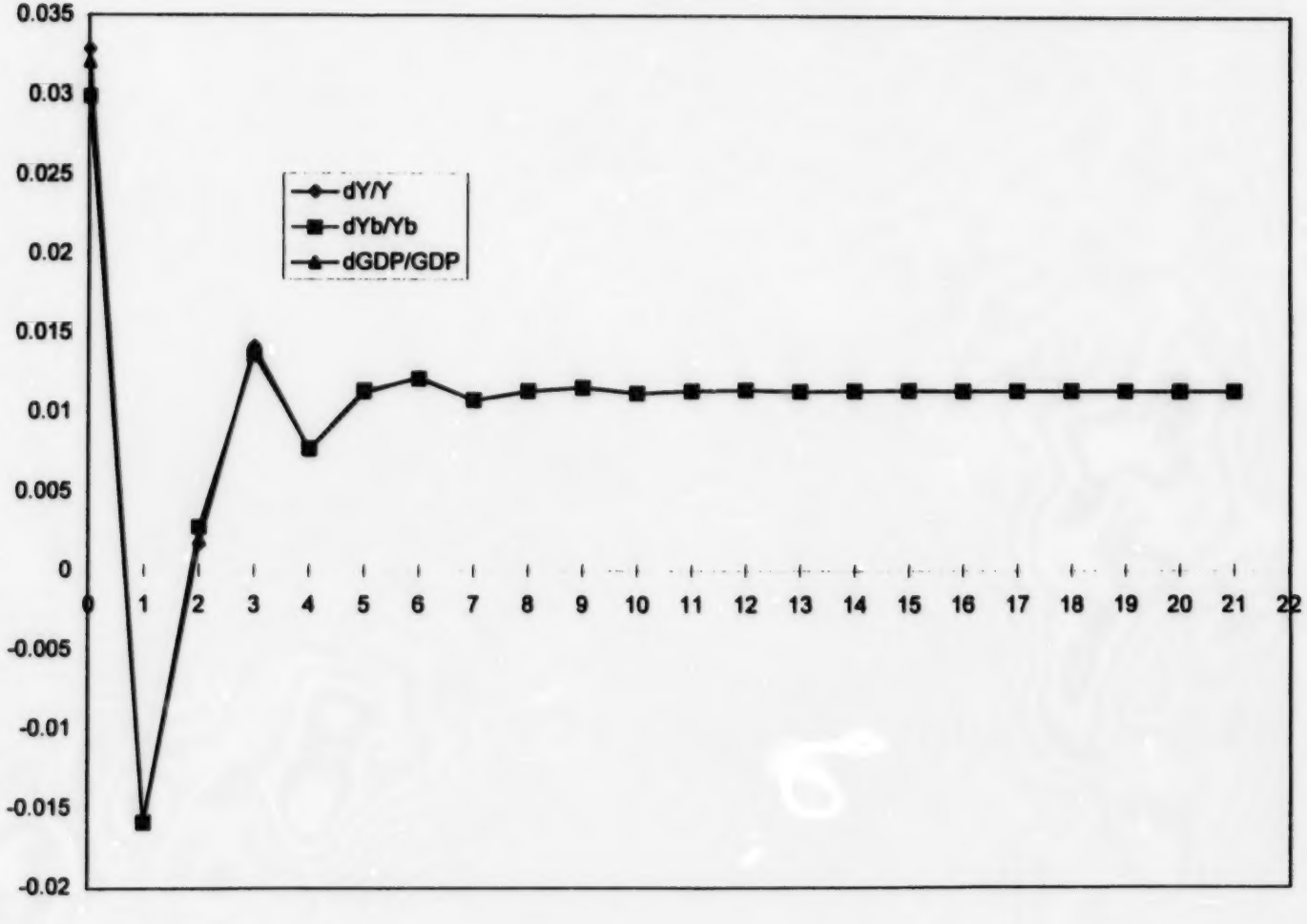

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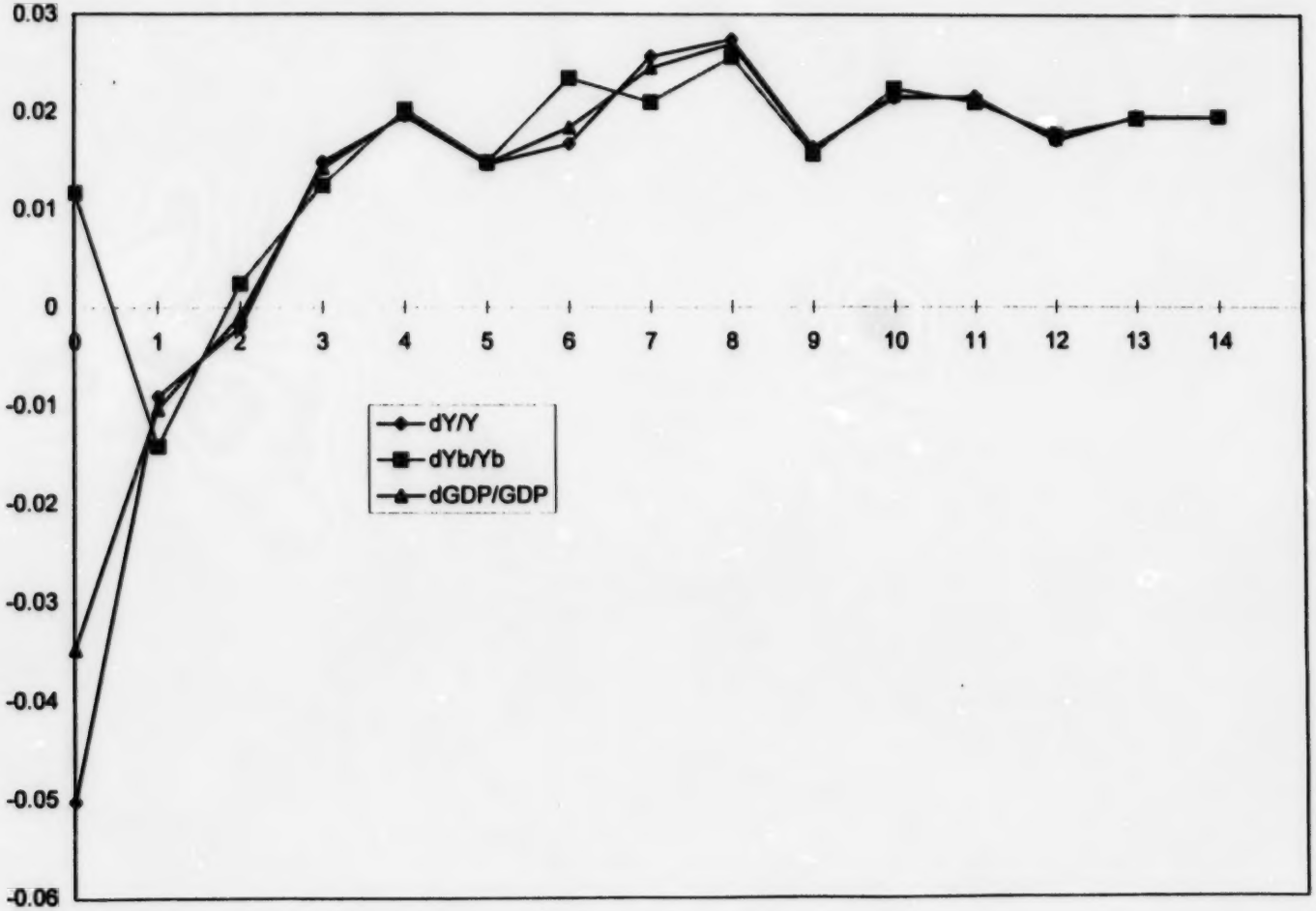

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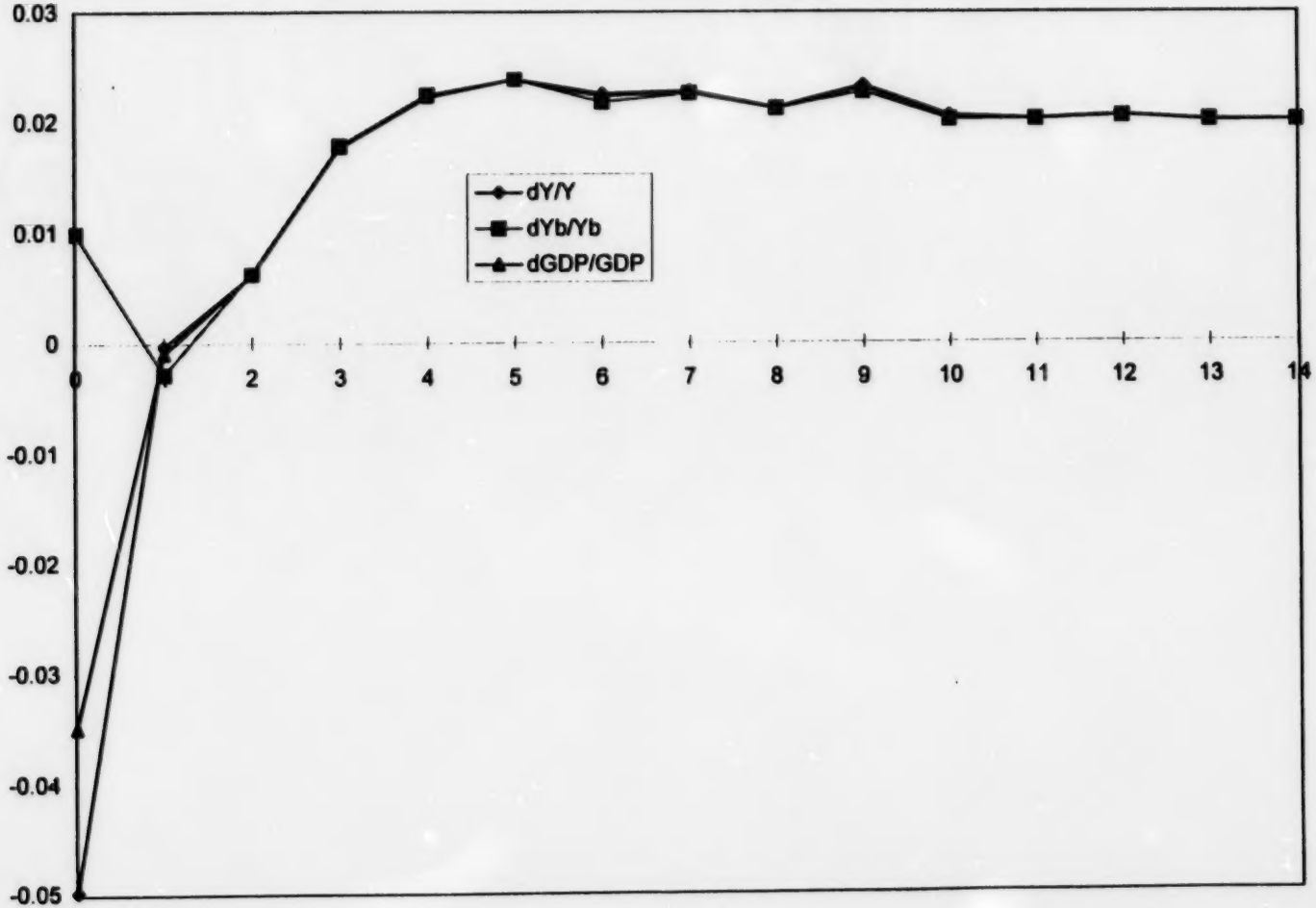
MODEL QT-HK



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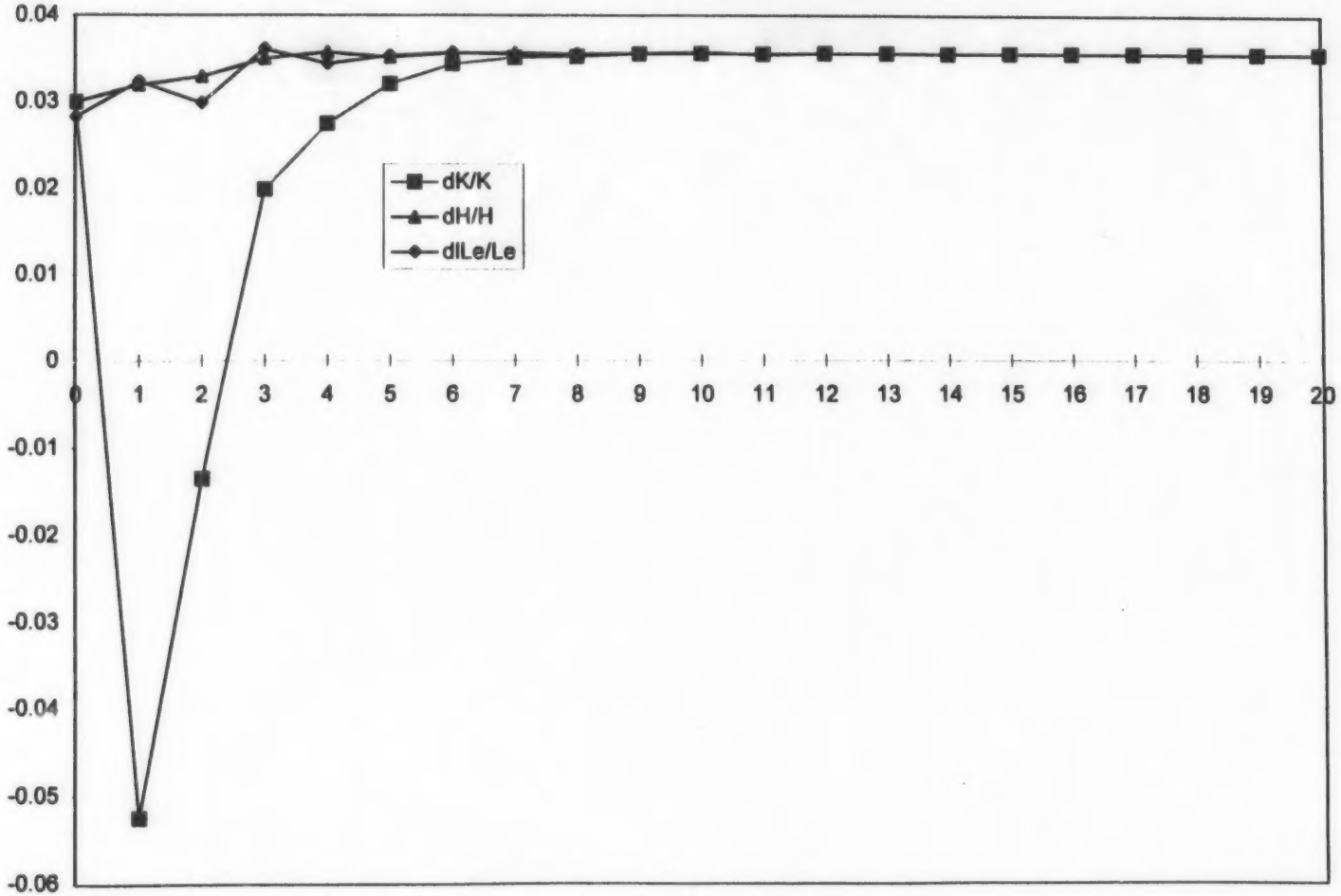

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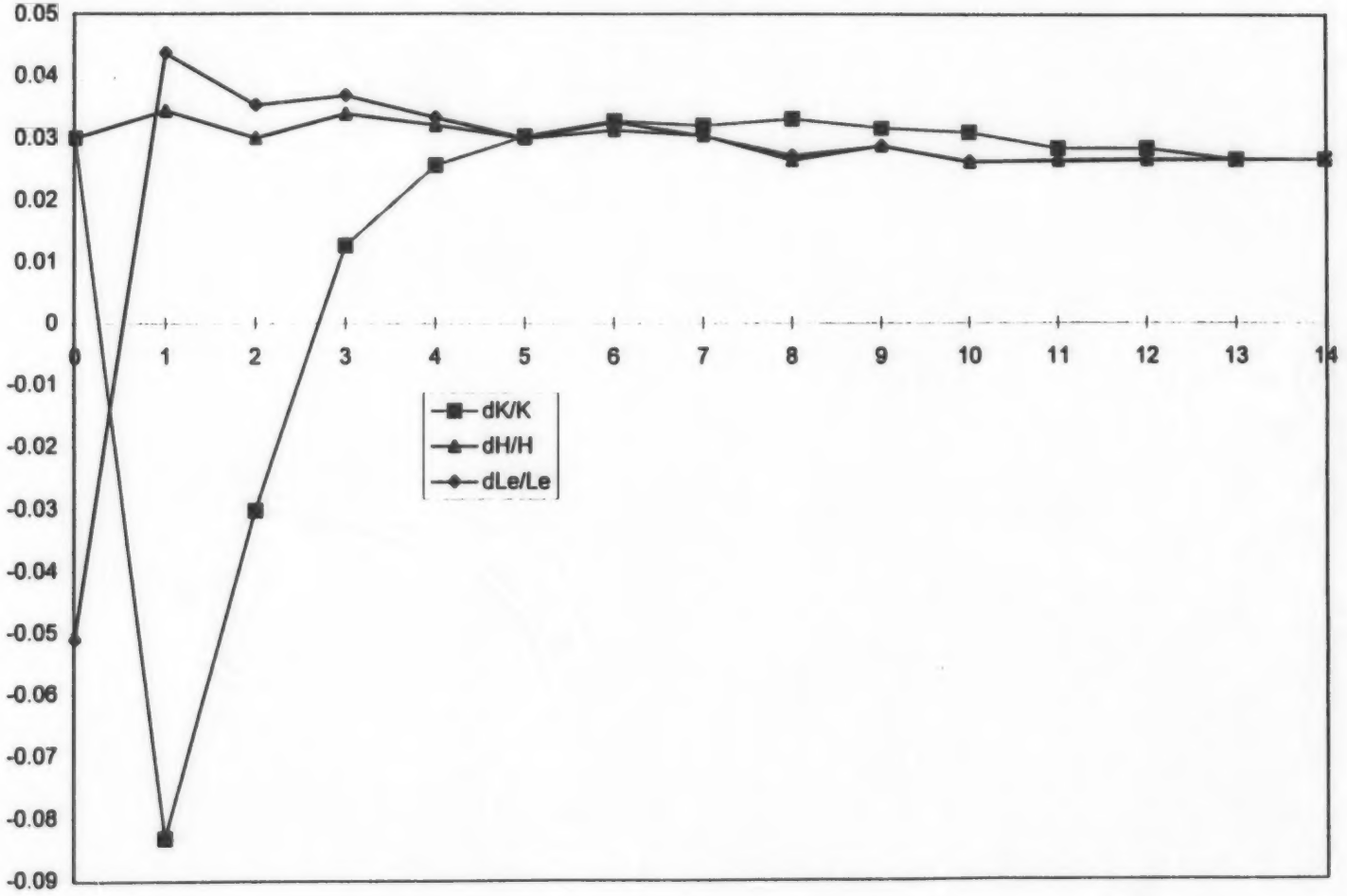
MODEL RT-H



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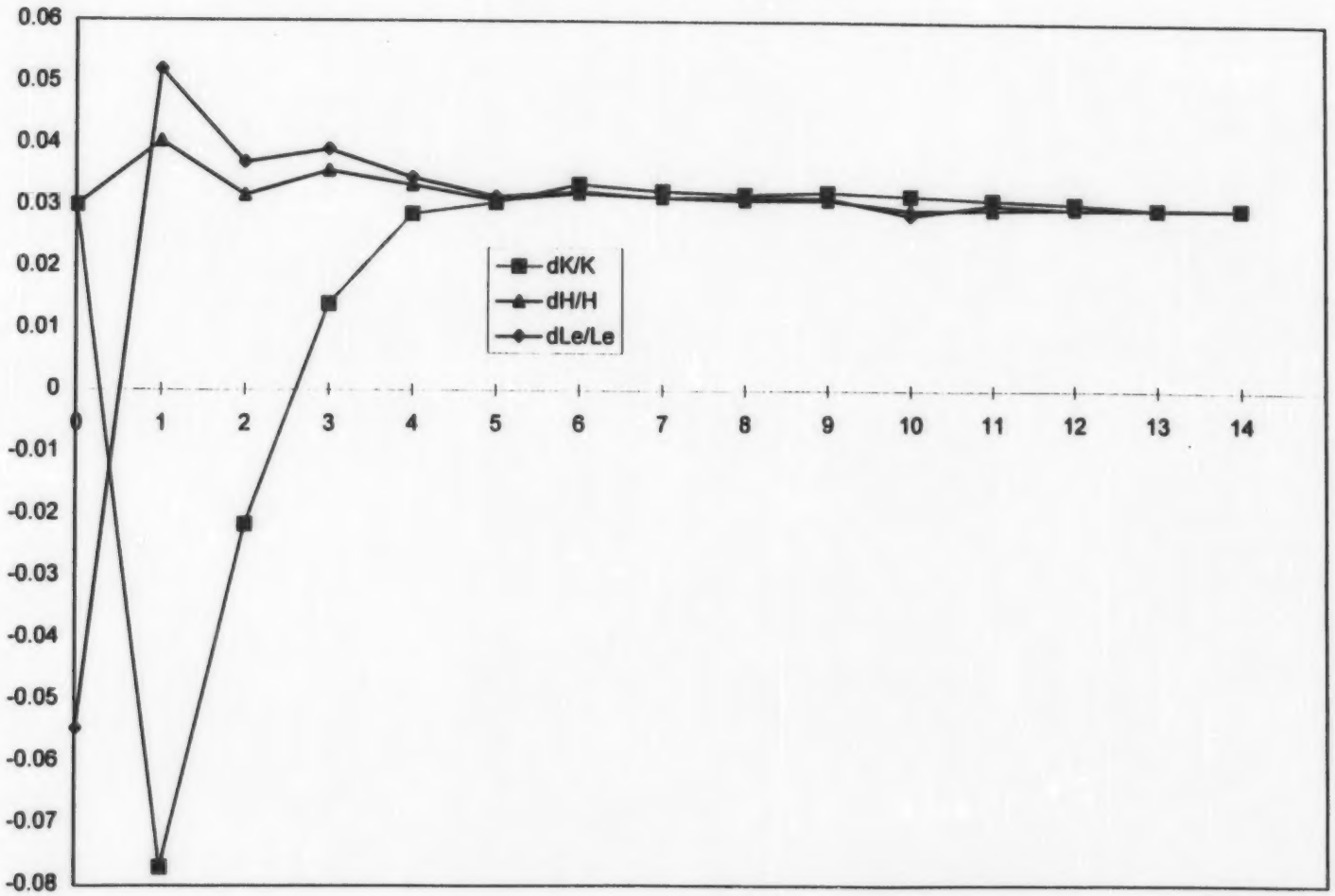
MODEL QT-H



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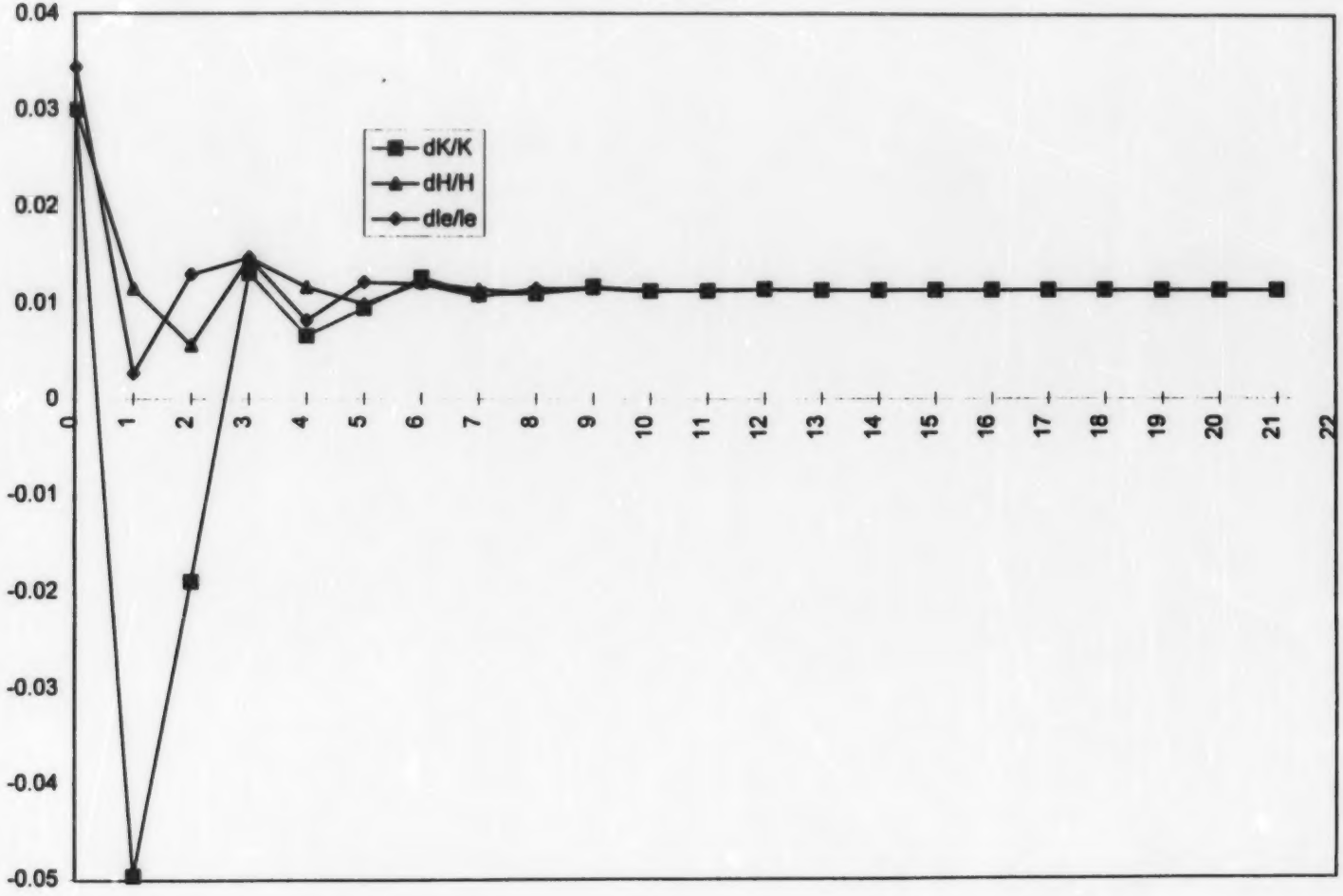

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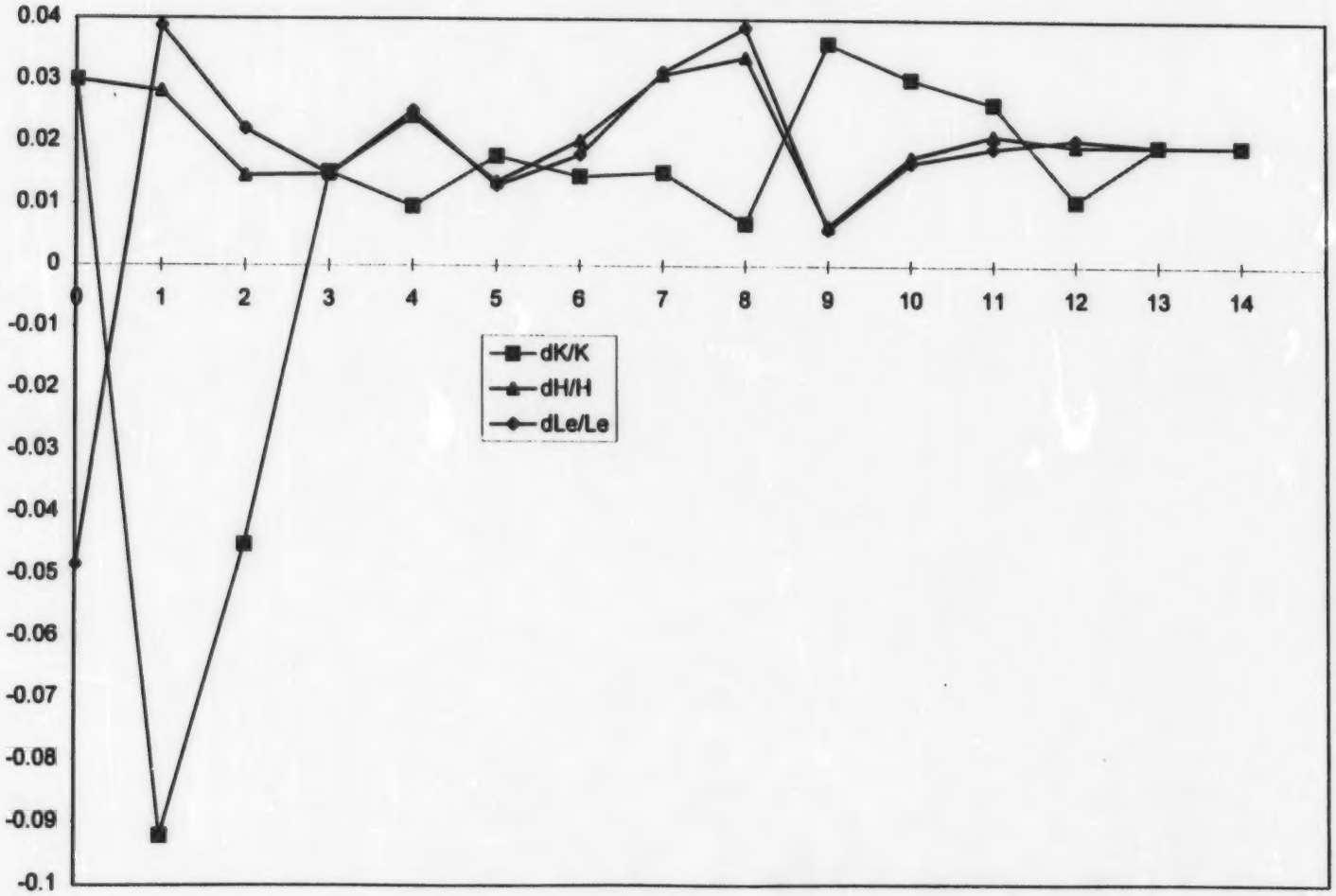

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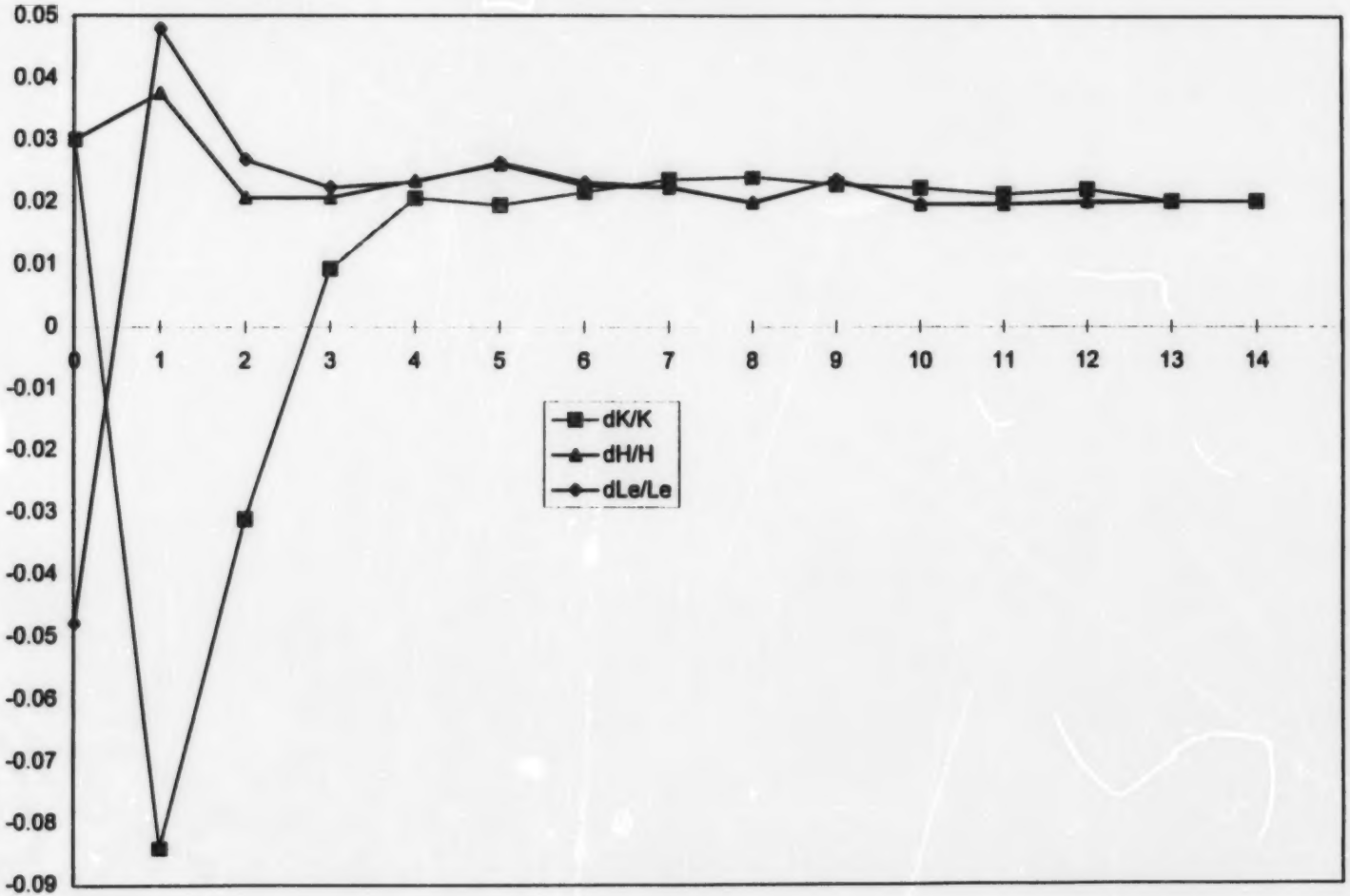
MODEL QT-HK



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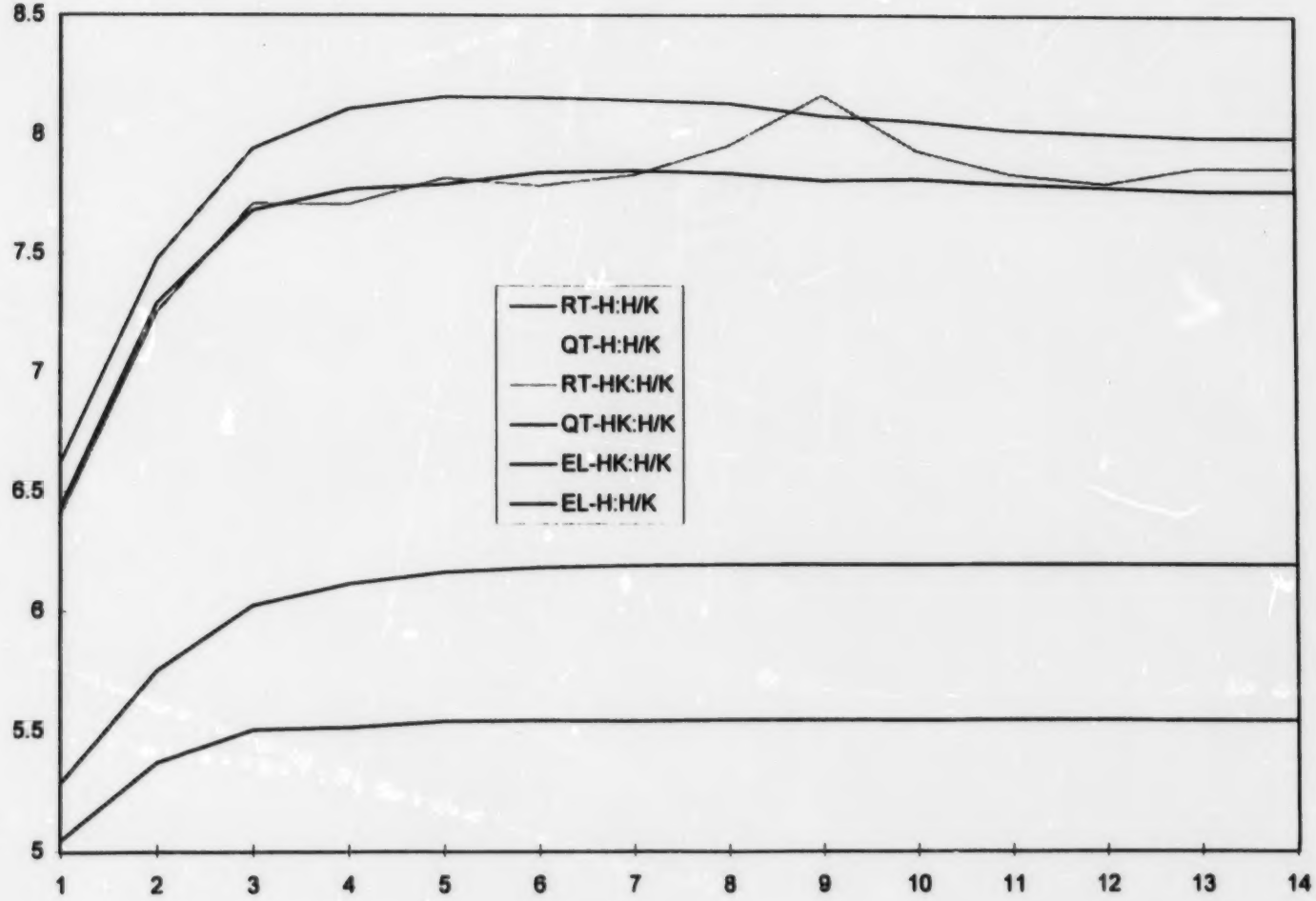
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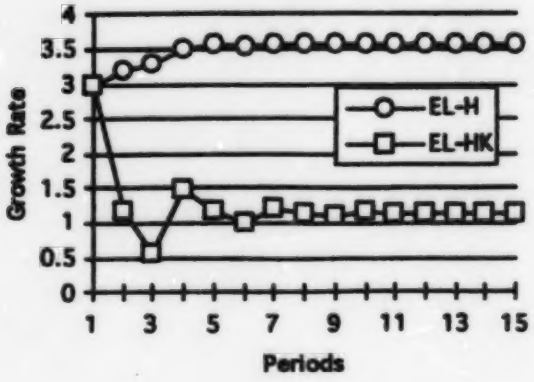
Figure 8



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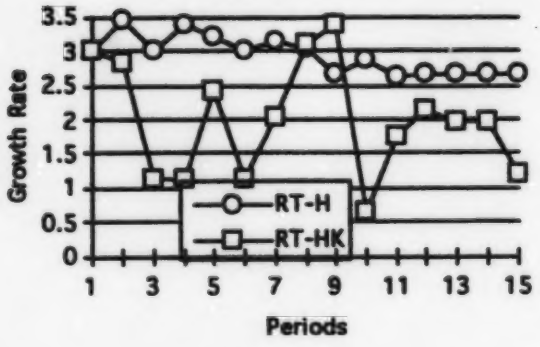

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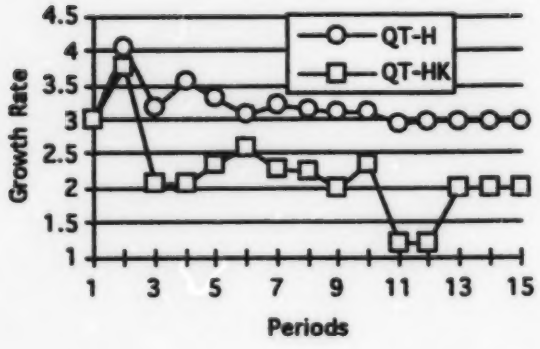

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